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MATHEMATICAL DESCRIPTION OF THE OMNIS SATELLITE ORBIT GENERATION PROGRAM (OrbGen)

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13. ABSTRACT (Maximum 200 words)

This document discusses the mathematical procedures used in the OMNIS orbit generation program. It provides a detailed description of the top-level procedures used for the numerical integration, references all of the force models employed and discusses those force models that have a substantial impact on the integration technique chosen. The document attempts to discuss the mathematics of the orbit integration but does not discuss the details of the implementation into code. A primary use of the program has been for Global Positioning System (GPS) satellites; however, it is used for low altitude satellites as well. Force models follow the World Geodetic System 1984 (WGS 84) constants and models, the International Earth Rotation Service (IERS) conventions when models are not covered by WGS 84, and specialized models when required, e.g., specialized radiation pressure forces for various GPS satellite vehicle types, as well as atmospheric density models for use in the atmospheric drag modeling of low altitude satellites. The program can segment drag forces, during the integration span, in order to accommodate inaccurate drag modeling. The program can accommodate several thrusts of arbitrary duration and magnitude. The inertial system used is J2000 and the conversion to the Earth-fixed International Terrestrial Reference Frame (ITRF) is based on the 1980 revision of the 1976 International Astronomical Union (IAU) precession and 1980 nutation theory, using the Earth Orientation Parameters published by the IERS. The program is completely coded in FORTRAN 77. It is coded in double precision and executes on a UNIX operating system.

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FOREWORD

This report discusses a numerical integration program used to calculate the ephemerides of satellites. It has descended from an earlier version in the sixties. In the sixties and seventies, NSWC determined orbits for the Navy Navigation Satellite System, TRANSIT, as well as for low altitude satellites. This work led to program changes in the seventies, allowing for more efficient orbit determination when faced with a high atmospheric drag environment. Also, in the seventies, NSWC was involved in the start-up of the Global Positioning System (GPS), and an earlier version of this program was selected as the GPS Master Control Station's (MCS) onsite integrator. Subsequently, NSWC provided the MCS with reference ephemerides for all of the GPS satellites from 1974 until 1986. In the eighties, the program was rewritten to upgrade the code and change some aspects of the integration algorithm itself. In the nineties, the program was changed to accommodate discontinuities caused by maneuvering satellites and also caused by solar radiation shadow boundaries encountered by the GPS satellites. In post 2000, the program was changed to incorporate the effects of Earth tides and planetary forces. This report discusses the mathematics underlying these and other environmental factors defining and influencing the structure of the numerical integration of orbits for Earth satellites.

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ILLUSTRATIONS

1 INTRODUCTION

This document discusses the mathematical procedures used in the OMNIS orbit integration program, OrbGen. It provides a detailed description of the top level procedures used for the numerical integration, references all of the force models employed, and gives some discussion of those force models which have a substantial impact on the integration technique chosen. The document attempts to discuss the mathematics of the orbit integration but does not discuss the detail of the implementation into code.

A primary use of the program has been for Global Positioning System (GPS) satellites; however, it is used for low altitude satellites as well. This use is primarily in association with the OMNIS Multisatellite Filter/Smoother [Swift-1], the OMNIS Corrector/Editor [Carr], and the OMNIS Batch Processor system of programs.

Force models follow the World Geodetic System 1984 [WGS84] constants and models, the International Earth Rotation Service (IERS) 1996 conventions [IERS96] when models are not covered by WGS84, and specialized models when required, e.g., specialized radiation pressure forces for various GPS satellite vehicle types, as well as atmospheric density models for use in the atmospheric drag modeling of satellites. *http://earth-info.nima.mil/GandG/tr8350_2.html* and *http://maia.usno.navy.mil/conventions.html*

Earth gravity is generally represented by EGM96 or, in the case of GPS, a small subset thereof. However, Earth gravity files of unnormalized coefficients can be accommodated at any size. Other force modeling is included for ocean tides, solid Earth tides, vehicle thrusting, and momentum dumping.

Thrusting, momentum dumping, radiation pressure modeling for shadow boundary transition, and drag segment transition all involve discontinuities that occur during an otherwise smooth orbit integration regimen. These discontinuities are all handled via a process of stepsize reduction and numerical integration restarting. Thrusts can be accommodated at any time and of any duration and magnitude.

Earth orientation is generally modeled with coefficients generated by the National Geospatial-Intelligence Agency (NGA), although any daily table, such as predicted or final values generated by the United States Naval Observatory (USNO) [IERS96], can be used as well.

Output ephemeris files can contain various combinations of the inertial and Earth-fixed position and velocity values, radiation model shadow transition times, and Earth orientation values. There can be orbit initial conditions and perturbation transformations relating how the ephemeris at any time would change due to a change in the initial conditions.

The inertial frame used is that of the mean equator and equinox of J2000.0 whose pole is closely aligned with the International Celestial Reference System (ICRS) pole, the Celestial Ephemeris Pole (CEP). Conversion to the Earth-fixed International Terrestrial Reference Frame (ITRF) is based on the 1980 revision of the 1976 International Astronomical Union (IAU) precession and 1980 nutation theory, using the Earth Orientation Parameters published by the International Earth Rotation Service (IERS) [IERS96].

The Earth Orientation Parameters of $UT1-UTC$, Δx, Δy, $dpsi$, and $deps$ are used, along with the precession-nutation theory, to relate the International Terrestrial Reference System to the ICRS. The celestial pole offset quantities, $dpsi$, $deps$, are observed corrections to the adopted precession-nutation model. The parameters Δx, Δy are measured offsets of the CEP from the IERS Reference Pole (IRP).

The basic mission of this program is to solve a family of ordinary differential equations (Equations of Motion) using a specific numerical technique (Cowell Coefficients). The procedure must be started (Starting Procedure) and it must continue integrating (Running Procedure). It must do this to obtain both the satellite orbit and the perturbation/variation transformations, which relate perturbations in the orbit at any time to changes in the initial conditions. The time history of these perturbation transformations are a solution of an associated set of equations to the equations of motion and can easily be considered to be just more equations in the original family.

Having started and now continuing to integrate, the procedure must next address how to restart (Integration Restart) if discontinuities of some type are encountered, e.g., solar radiation shadow boundaries.

The numerical procedure requires evaluating the accelerations that come from the forces that act on the satellite. This in turn often requires calculating certain transformations, e.g., Inertial Earth-fixed transformation, which allows for calculating the gravity force in the Earth-fixed frame and converting the answer to the inertial frame. The inertial frame is the frame in which the family of differential equations is presented.

File structures are listed in order to give a better picture of where the input comes from, where the output goes, and what the database values are that forces and transformations need for their evaluation.

For example, the Input Data file shows that initial conditions can come in the form of rectangular coordinates or osculating orbital elements (classical or special format) in degrees or radians. It shows that atmospheric drag scaling parameters can be redefined twenty times during the integration span, thrust components can be redefined four times, and momentum components, ten times. The file also shows that initial conditions can come from various places, e.g., an Initial Conditions file, an existing Satellite Trajectory file, or the Input Data file itself. Any of the above values on the Input Data file will always override the same values from another source.

The Satellite Trajectory file shows that there are eight extra time lines at the beginning and the end of the file, which are used for the Lagrange interpolation procedure. These values allow other associated trajectory files to be written with four extra time lines at each end. This in turn allows for getting points from these files that are very near the beginning or end of the file. A typical associated trajectory file might be one obtained by using the stored perturbation transformations to write a trajectory that differs from the original one by perturbing the initial conditions. Another might be obtained by perturbing the points on the original trajectory through some orbit estimation procedure, e.g., a Kalman Filter/Smoother data processing procedure.

The interpolation procedure is also used to obtain initial values from an existing trajectory file,

when that option is chosen for starting an integration. When starting at a time other than the epoch of an existing trajectory file, the interpolation procedure will return interpolated velocities, as the trajectory file only stores inertial velocity values at epoch. The tabulated inertial values on the trajectory are only position values with velocity values obtained via interpolation. The interpolation procedure uses the derivative of the Lagrange interpolation formulas to obtain velocity values at other than the epoch time. This means, for example, that one can not reproduce a satellite orbit to all digits, if starting from other than epoch, even though no other model changes have been made, if the initial conditions are obtained by interrogating an existing trajectory.

To properly carry out such a procedure, the original integrated inertial velocity values would have to be obtained by some other means, e.g., having saved them during a previous integration or converting the stored Earth-fixed velocity values.

Each of the above topics form the subject matter in the following chapters.

The program is completely coded in FORTRAN 77. It is coded in double precision and executes on a UNIX operating system.

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2 EQUATIONS OF MOTION

The satellite equations of motion can be expressed in the form

$$\begin{aligned}\ddot{x} &= A_{earth}(x,P,t) + A_{sun}(x,t) + A_{moon}(x,t) \\ &+ A_{planet}(x,t) + A_{tide}(x,P,K_l,t) + A_{drag}(x,\dot{x},C_d,t) \\ &+ R_{thrust}(x,\dot{x})A^{body}_{thrust}(T,t) + R_{radiation}(x)A^{body}_{radiation}(x,Kr,t) \\ &= G(x,\dot{x},P,C_d,T,Kr,t)\end{aligned} \tag{2.1}$$

A_{earth} is the acceleration due to Earth gravity. Earth gravity is modeled as a spherical harmonic expansion with the option of using any gravity field, such as the Earth Gravity Model of 1996 [EGM96], a joint development of NGA and the National Aeronautics and Space Administration (NASA). The accelerations A_{sun}, A_{moon}, and A_{planet} are modeled as point masses.

A_{tide} represents solid-Earth lunar and solar tidal accelerations. There is an option for using a second-order Legendre polynomial representation with a Love's number multiplier, K_l, termed the NSWCDD model, or for using an implementation of solid-Earth tides as presented in the 1996 IERS conventions [IERS96]. In addition $A_{poletide}$ and $A_{oceantide}$ accelerations are included in A_{tide}, with both implementations following [IERS96].

The Earth orientation parameters P consist of polar motion, Δx, Δy, and Earth rotation $(UT1 - systemtime) = \Delta t_2$. The *systemtime* is generally GPS time. These values are part of the inertial-to-Earth-fixed rotation transformation, but the default condition is for only Δt_2 to be used in the gravity field model/acceleration rotation between Earth-fixed and inertial space.

A_{drag} is acceleration due to atmospheric drag, with C_d being a time-dependent set of scaling constants, which are constant on the subintervals (maximum of 20) of a partition of the integration time span. An atmospheric density option allows for using an exponential density model developed at NSWCDD [Smith], or the Jacchia-Bass 1977 model [Jacchia], [Bass]. The Jacchia model requires the use of a current Geophysical Data Base of solar flux values and geomagnetic indices. The Barlier Drag Temperature Model [Barlier-1] has been implemented, but is not currently part of the operational program. Other DTM models such as [Barlier-2] and more recent versions from 1994 and 2000 are available.

A^{body}_{thrust} is acceleration due to thrust in a body frame, with T providing three constant acceleration values in that frame, applied over specific time intervals (maximum of 4). R_{thrust} transforms the body frame acceleration to the inertial frame.

$A^{body}_{radiation}$ represents the solar radiation force in a body frame with $R_{radiation}$ transforming to the inertial frame. Options exist for using a simple spherical satellite model for solar radiation and for using several GPS specific models. These models are specific to the type of GPS satellite, such as Block I, Block II, Block II A, or Block II R. Currently only the Block II A and the Block II R satellites are being processed. Options exist to use official Air Force Operational Control Segment (OCS) models for Block II A and Block II R, Jet Propulsion Laboratory (JPL), and Aerospace Corporation models. These models are documented in [Moore], [Porter], [Fliegel], [Gallini], [Block II R], and [Bar-Sever-1].

In each case, the radiation force is presented in a satellite body frame, the components are scaled by parameter, Kr_1, a y-axis (body frame) force Kr_2 is added, and parameter Kr_3 is seldom used. These force components are then scaled by a factor, *shape*, with value between zero and one. This is designed to account for the decreased amount of solar pressure during periods when the satellite is in partial shadow or prenumbra ($0 < shape < 1$). Full shadow or umbra implies that $shape = 0$. This shadow condition is caused by a combination of the Earth and the Moon obscuring the satellite's view of the Sun. The resultant force, computed in the satellite body frame, is then transformed to the inertial frame.

Initial values for the equations of motion consist of inertial position and velocity, a preferred set of satellite osculating orbital elements derived from classical elements,

$$\begin{aligned} e_1 &= a & e_4 &= i \\ e_2 &= e\sin(\omega) & e_5 &= l+\omega \\ e_3 &= e\cos(\omega) & e_6 &= \Omega \end{aligned} \tag{2.2}$$

or the classical orbital elements given below. Elements can be in radians or degrees. The values themselves can come from an initial conditions file, an existing trajectory file, or an input data file. Any input values from an input data file will override values obtained in another way. This is true for radiation, drag, and thrust input as well.

In either case, the initial values will be represented here by x_0 and $\dot{x}_0$. In addition there are model parameters for drag and thrust with values defined by the given partitions, and for radiation, tide, and polar motion. Earth gravity and tides have coefficients, which are maintained in a database.

$$\begin{aligned} a &= \text{semimajor axis} & i &= \text{inclination} \\ e &= \text{eccentricity} & l &= \text{mean anomaly} \\ \omega &= \text{argument of perigee} & \Omega &= \text{right ascension} \end{aligned} \tag{2.3}$$

Initial orbits are generally improved via data analysis using least squares or Kalman filtering techniques. Also, these techniques are used for subsequent data analysis using refined orbits. This generates a need for orbit perturbation information, in the form of partial derivatives with respect to initial osculating orbital elements or position and velocity and some or all of the model parameters used in generating an orbit.

Partial derivatives are generated by integrating orbital variational equations simultaneously with the orbit. The equations for this work are given by

$$\ddot{q} = (\partial G/\partial x)q + (\partial G/\partial \dot{x})\dot{q} \tag{2.4}$$

$$\ddot{q} = (\partial G/\partial x)q + (\partial G/\partial \dot{x})\dot{q} + (\partial G/\partial p) \tag{2.5}$$

The solutions to (2.4) for orbital element partials and (2.5) for model parameters, p, from (P, C_d, T, Kr) are given by

$$\psi = \psi_{x_0\dot{x}_0} = \partial x(t)/\partial x(0)\dot{x}(0) \quad \psi_p = \partial x(t)/\partial p \tag{2.6}$$

$$\dot{\psi} = \dot{\psi}_{x_0\dot{x}_0} = \partial \dot{x}(t)/\partial x(0)\dot{x}(0) \quad \dot{\psi}_p = \partial \dot{x}(t)/\partial p \tag{2.7}$$

Initial values for $\psi_{x_0\dot{x}_0}$, $\dot{\psi}_{x_0\dot{x}_0}$, ψ_p, and $\dot{\psi}_p$ are given by the identity if $(x_0, \dot{x}_0)$ represents position and velocity at the trajectory epoch or are derived by differentiating the equations relating position and velocity with the preferred orbital element set (2.2). Initial values for the model parameter partials are zero at epoch. If the integration restarts at a later time then all partials would be initialized at values that were current at that time.

Since the structure of the variational equations (2.4) and (2.5) is the same as the orbit equations (2.1), the equations $\ddot{x} = G(x, \dot{x}, p, t)$ will represent all of the equations and the solution $x(t) = x(t, x(0), \dot{x}(0), p)$ will stand for the time dependent solution to all differential equations, unless otherwise indicated.

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3 COWELL COEFFICIENTS

The integration technique used is the Gauss-Jackson method which is taken here to mean that the second-order acceleration equations (2.1), (2.4) and (2.5) are integrated directly without using any reference orbit and without converting them to first-order equations. The technique uses backward differences of accelerations with first and second sums, rather than backward accelerations together with position and velocity. The resulting formulas for calculating current position and velocity are presented as a linear combination of backward differences, at equally spaced points, together with first and second sums. The coefficients in the linear combination are termed "Cowell Coefficients". This method is described in [Jackson], [Herrick], [Merson], [Lundberg], and [Fox].

The formulas below provide for the calculation of the Cowell Coefficients. They begin with an initialization and then give two forms for the calculation. One form is used when coefficients must give an answer for a time point located at one of the equally spaced points. The other form is used for answers at arbitrary time points.

Initialization

$$
\begin{aligned}
&b(0,-2) = 0 \qquad b(0,-1) = 1 \\
&b(0,k) = -\sum_{j=1}^{k+1} b(0,k-j)/(1+j) \qquad k \geq 0 \\
&c(0,k) = \sum_{j=0}^{k+2} b(0,j-1)b(0,k+1-j) \qquad k \geq -2
\end{aligned}
\tag{3.1}
$$

which gives

$$
c(0,-2) = 1 \qquad c(0,-1) = -1
$$

Calculation at one of the equally spaced points (s, an integer)

$$
\begin{aligned}
&b(s,-2) = 0 \qquad c(s,-2) = 1 \\
&b(s,k) = b(s-1,k) - b(s-1,k-1) \qquad k \geq -1 \\
&c(s,k) = c(s-1,k) - c(s-1,k-1) \qquad k \geq -1 \\
\text{e.g.,}\quad &b(s,-1) = 1 \qquad c(s,-1) = -(1+s)
\end{aligned}
\tag{3.2}
$$

Calculation at an arbitrary point (s, real)

$$
\begin{aligned}
&d(s,0) = 1 \\
&d(s,k) = d(s,k-1)(k-1-s)/k \qquad k \geq 1 \\
&b(s,k) = \sum_{j=0}^{k+2} d(s,j)b(0,k-j) \qquad k \geq -2 \\
&c(s,k) = \sum_{j=0}^{k+2} d(s,j)c(0,k-j) \qquad k \geq -2
\end{aligned}
\tag{3.3}
$$

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4 STARTING PROCEDURE

During the start procedure the drag, thrust, and radiation conditions in effect at the trajectory epoch, T_0, are generally maintained throughout the start. This means that no changes to the drag, thrust, or radiation profiles are made during start. The profiles are defined by parameter values, and, thus, the values should not be changed during start. In particular, the value of the shadow-related *shape* function in section 7.3, page 20, used to describe the shadow condition of the radiation model, is held constant and equal to the value at the starting time point T_0. If however, the *shape* function has a value which is not 0 or 1, then radiation is in transition, going through the prenumbra, and the functional value of *shape*, SHAPET, should be used at each time step, together with the reduced step h_1. Shadow testing, as described in subsection 7.3, page 20, must be carried out as usual, but no restarting is permitted during the start routine.

A. Absolute time, T, is related to integration time, t, by $T = T_0 + t$ in forward integration and by $T = T_0 - t$ in backward integration, where T_0 is the starting epoch time. Position at time T will be denoted by $x(t)$ and $y(t)$, respectively. In addition, backward integration requires the program to go through the initialization described in section 9, page 33 BACKDATE, and to evaluate the forces at the appropriate absolute (year, day, second) time corresponding to T. During backdate, the correct time from epoch T_0 is computed by $t_{backdate} = 2t_f - t$ and the correct absolute time to use in evaluating forces is given by $T = T_0 + t_{backdate}$. The starting conditions are given at a time, t_f, measured from epoch. Thus, when starting at epoch, $t_f = 0$ and $t_{backdate} = -t$.

The algorithms presented are for forward integration. Initialization procedures for backward integration are given and backward integration is carried out with time moving forward, just as in forward integration. Each time that forces are calculated which require absolute time, such as Earth rotation or the evaluation of Sun and Moon position, care must be taken to use $t_{backdate}$ rather than t in calculating T.

Initial values are calculated by Taylor expansion for $x, \dot{x}, \ddot{x}$ at times, t, given by $t = t_f + t_d - kh, \quad k = 0, \ldots, i-1$. The value, $N = i - 1$, is the order of integration. h is the integration time step, either the input value h_0 or an input reduced value h_1. h_1 could be the reduced value for radiation shadow boundary transition or the reduced value for thrust. The starting time, t_f, is measured from epoch, T_0, and is the time at which the initial conditions are provided. This time is generally zero but would be different than zero at a restart time. The point $x(t_f)$ is called the point held fixed and does not change during the starting procedure. The value t_d is the time of the difference table measured from t_f. The time of the difference table is given by $t_d = rh$, where $r = 0$ for the standard integration procedure and $r = [i/2]$ for the central integration procedure. [ ] is the greatest integer function. The initial position and velocity at epoch, T_0, will come from input initial conditions, but will also be obtained during integration, either from input or calculation, if there is a need to use the starting routine for restarting at a $t_f \neq 0$. Examples of this situation include drag or thrust boundaries determined from input values, or radiation shadow boundaries determined dynamically during the integration procedure.

$$h = h_0 || h_1 \tag{4.1}$$
$$t_d = rh \tag{4.2}$$
$$t = t_f + t_d - kh \tag{4.3}$$
$$t_{backdate} = 2t_f - t \tag{4.4}$$
$$\ddot{x}_f = G(t_f, x_f, \dot{x}_f) \tag{4.5}$$
$$x(t) = x_f + (t - t_f)\dot{x}_f + (t - t_f)^2 \ddot{x}_f / 2 \tag{4.6}$$
$$\dot{x}(t) = \dot{x}_f + (t - t_f)\ddot{x}_f \tag{4.7}$$
$$\ddot{x}(t) = G(t, x(t), \dot{x}(t)) \tag{4.8}$$

$x_f, \dot{x}_f, \ddot{x}_f, h$ and G represent the initial position, velocity, acceleration, integration step and and total acceleration function, respectively. The function G must be evaluated at the correct absolute (year, day, second) time.

B. Given i accelerations $\ddot{x}(t_f + t_d - kh), \quad k = 0, \ldots, i-1$ and a position and velocity $x(t_f), \dot{x}(t_f)$, form a backward difference table at $t_f + t_d$. Denote $t = t_f + t_d - kh$ by t_k and $x(t)$ by x_k. This gives $t_f + t_d = t_0$ when $k = 0$. Define the backward differences by

$$\nabla^0 \ddot{x}_k = \nabla^0 \ddot{x}(t) = \ddot{x}(t_d - kh) = \ddot{x}_k \tag{4.9}$$
$$\nabla^j \ddot{x}_k = \nabla^{j-1} \ddot{x}_k - \nabla^{j-1} \ddot{x}_{k+1} \qquad j > 0 \tag{4.10}$$

B.1 Using the backward differences from B, calculate first and second sums at t_0 by

$$\nabla^{-1} \ddot{x}_0 = \dot{x}(t_0 - sh)/h - \sum_{p=0}^{i-1} b(s,p) \nabla^p \ddot{x}_0 \tag{4.11}$$

$$\nabla^{-2} \ddot{x}_0 = x(t_0 - sh)/h^2 + (1+s) \nabla^{-1} \ddot{x}_0 - \sum_{p=0}^{i-1} c(s,p) \nabla^p \ddot{x}_0 \tag{4.12}$$

$x(t_0 - sh)$ is the point held fixed, $x(t_f)$, in defining the first and second sums. The value of s is usually r, where $r = 0$ in the standard integration procedure and $r = [i/2]$ in the central procedure.

C. Use the difference table to calculate new values $(x, \dot{x}, \ddot{x})$ at $t = t_0 - kh$ for values of $k = 0, \ldots, i-1 = N$.

$$x(t) = x(t_0 - kh) = x_k \tag{4.13}$$

$$x_k = h^2 \left[\nabla^{-2} \ddot{x}_0 - (1+k) \nabla^{-1} \ddot{x}_0 + \sum_{p=0}^{i-1} c(k,p) \nabla^p \ddot{x}_0 \right] \tag{4.14}$$

$$\dot{x}_k = h \left[\nabla^{-1} \ddot{x}_0 + \sum_{p=0}^{i-1} b(k,p) \nabla^p \ddot{x}_0 \right] \tag{4.15}$$

$$\ddot{x}_k = G(t, x(t), \dot{x}(t)) \tag{4.16}$$

D. For each component of $x, \dot{x}$, compute for each line, the absolute difference between the current value and the corresponding value from the previous iteration. If all of the values are less than their appropriate input tolerance $(Tolx, Tol\dot{x})$, then the starting procedure is converged. If the procedure is not converged, then return to step B and recompute differences using the new $\ddot{x}$ values. Continue in this way until convergence is reached. Save the difference table at t_0 and the accelerations

$$\ddot{x}(t_0-(i-1)h), \ddot{x}(t_0-(i-2)h), \ldots, \ddot{x}(t_0) \tag{4.17}$$

If the forward integration route is being used, then continue to step E, and if the backward integration route is in use, then go to section 5, page 15, RUNNING PROCEDURE.

E. Form a difference table at $\tau_0 = (i-1)h - t_0$, in preparation for backward integration. The fixed point at $\tau_f = \tau_0$ is:

$$(y(\tau_0), \dot{y}(\tau_0)) = (x(t_0-(i-1)h), -\dot{x}(t_0-(i-1)h)) \tag{4.18}$$

and the backward accelerations are given by:

$$\ddot{y}(\tau_0 - kh) = \ddot{x}(t_0-(i-1)h+kh) \qquad k = 0, 1, \ldots, i-1 \tag{4.19}$$

Drag coefficients, appropriate thrust coefficients, and angular velocity are negated as described in the initialization procedure of section 9, page 33, BACKDATE.

The calculations in B and B.1 can now be used to form a difference table with $r = 0$.

F. Use the difference table at τ_0 and the backward route in section 5, RUNNING PROCEDURE, with $s = 0$, to obtain additional negative time lines. When the number of time lines equals the input value necessary for interpolation, then backward timeline trajectory values are written in increasing time order. Velocity values are to be negated before writing. Values are written at the prescribed write step, h_w, until the write time passes epoch t_f. When this point is reached the RUNNING PROCEDURE is entered, with the difference table at t_0 which was saved in step D, and the forward integration route is used.

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5 RUNNING PROCEDURE

This procedure is occasionally interrupted by restarts caused by events such as segment boundaries for drag, thrust, or momentum. And by encountering solar radiation related Earth shadow boundaries. When this occurs the integration time line may change from a multiple of the integration time step to an arbitrary value dynamically determined by the particular event. The impact of this is covered in part H below, for in this case s is not zero.

A. Extrapolate the difference table, at $t-h$, forward to time t by

$$\nabla^{i-1}\ddot{x}(t) = \nabla^{i-1}\ddot{x}(t-h) \tag{5.1}$$

$$\nabla^{p-1}\ddot{x}(t) = \nabla^{p}\ddot{x}(t) + \nabla^{p-1}\ddot{x}(t-h) \qquad p \leq i-1 \tag{5.2}$$

B. Calculate position and velocity by

$$x(t-sh) = h^2\left[\nabla^{-2}\ddot{x}(t) - (1+s)\nabla^{-1}\ddot{x}(t) + \sum_{p=0}^{i-1} c(s,p)\nabla^{p}\ddot{x}(t)\right] \tag{5.3}$$

$$\dot{x}(t-sh) = h\left[\nabla^{-1}\ddot{x}(t) + \sum_{p=0}^{i-1} b(s,p)\nabla^{p}\ddot{x}(t)\right] \tag{5.4}$$

where $s=0$, in general, but may be nonzero and also noninteger if values other than the values at an integration line are required. Such values include restart calculations for drag, thrust, momentum dumps (modeled as thrusts), and shadow boundary point crossings due to eclipsing in the solar radiation pressure model.

C. Calculate acceleration by

$$\ddot{p}(t-sh) = G(t-sh, p, \dot{p}) \qquad \text{where} \quad s=0 \tag{5.5}$$

Setting $p=x$ and $\dot{p}=\dot{x}$ allow p values to change and x values to remain fixed in C through F.

D. Calculate corrections by

$$\delta = \ddot{p}(t) - \nabla^{0}\ddot{x}(t) \tag{5.6}$$

E. Improve position and velocity at time t $(s=0)$ by

$$p(new) = x + h^2\delta \sum_{i=-2}^{i-1} c(s,l) = x + h^2\alpha\delta \tag{5.7}$$

$$\dot{p}(new) = \dot{x} + h\delta \sum_{i=-2}^{i-1} b(s,l) = \dot{x} + h\beta\delta \tag{5.8}$$

F. If the maximum iteration per time line has not been reached then go to step C. If the maximum iteration has been reached, then calculate the final position, velocity and corrected difference table.

G. Calculate the final position, velocity and difference table at time t by

$$x(new) = p \tag{5.9}$$

$$\dot{x}(new) = \dot{p} \tag{5.10}$$

$$\nabla^l \ddot{x}(new)(t) = \nabla^l \ddot{x}(t) + \delta \qquad l = -2, -1, 0, 1, 2, \ldots, i-1 = N \tag{5.11}$$

H. If $t - rh \geq t_w$, the next write time, then write the time, t_w, Earth-fixed position and velocity, inertial position, partial derivatives, and other data, as prescribed, to the output trajectory file. Use **B** to carry out these computations if $s \neq 0$. In **B**, the value of s for t_w is calculated from $t_w = t - sh$ giving $s = (t - t_w)/h$.

When s does not represent an integration node value, then the $d(s,j)$ coefficients from (3.3) COWELL COEFFICIENTS will be needed for the calculation of the $b(s,p)$ and $c(s,p)$ values in B. This procedure is continued until the write time t_w passes $t - rh$, i.e., $t_w > t - rh$.

6 VARIATIONAL EQUATION OPTION

If the maximum iterations per time line is greater than one then this option provides a procedure which involves iterating position and velocity only.

The iteration procedure should be followed in the standard way for the position and velocity, but the partials should not be iterated at all. After extrapolating the difference table forward in (5.1), an estimated position and velocity of the partial, q_{est}, $\dot{q}_{est}$, is calculated in (5.3). A final corrected position and velocity of the partial is now calculated by solving the equations:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} q_{est} + B_1 \\ \dot{q}_{est} + B_2 \end{bmatrix} \tag{6.1}$$

where

$$\begin{array}{lll} A_{11} = 1 - h^2 A\alpha & A_{12} = -h^2 B\alpha & B_1 = h^2(C - \nabla^0 \ddot{q}_{est}(t))\alpha \\ A_{21} = -hA\beta & A_{22} = 1 - hB\beta & B_2 = h(C - \nabla^0 \ddot{q}_{est}(t))\beta \\ A = \partial G/\partial x & B = \partial G/\partial \dot{x} & C = \partial G/\partial p \\ q_1 = q = \partial x/\partial p & q_2 = \dot{q} = \partial \dot{x}/\partial p & \end{array}$$

and α, β come from (5.7).

Calculate acceleration, $\ddot{q}$, from (5.5) using the final corrected position and velocity, q and $\dot{q}$.

$$\ddot{q} = Aq + B\dot{q} + C \tag{6.2}$$

Calculate the correction term, δ, in (5.6).

$$\delta = \ddot{q}(t) - \nabla^0 \ddot{q}_{est}(t) \tag{6.3}$$

For each value of l correct the difference table using (5.9).

$$\nabla^l \ddot{q}(t)(new) = \nabla^l \ddot{q}_{est}(t) + \delta \qquad l = -2, -1, 0, 1, 2, \ldots, i-1 = N \tag{6.4}$$

Write the solution, $\Psi_p = q$, on the trajectory file, where p is a member from the set of orbit and model parameters.

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7 SHADOW RESTART

After converging at time line t, if shadow testing in section 7.3, page 20, indicates that the shadow has been entered at $t-rh$, but that it had not yet been entered at $t-(r+1)h$, then the shadow flag, ISHDOW, will have been set to TRUE and the shadow boundary time, t_p, will be determined. This condition can occur by entering the shadow region from a period of sunlight or a period of no sunlight called the umbra.

7.1 Entering Shadow

Determine the shadow boundary time, t_p, by setting $a=t-rh-h$, $b=t-rh$, where t_p is within this interval. Compute the midpoint, $m_p=(a+b)/2$. Redefine "a" and "b" in such a way that the new interval, $[a,b]$, will be $[a,m_p]$ or $[m_p,b]$. Choose the new "a" and "b" values such that the *shape* value at the new "a" is 1(0) and the *shape* value at the new "b" is less (greater) than 1(0). Continue this process until $b-a$ is less than a tolerance on the order of 10^{-6}. Use (5.3) to calculate the various $x(m_p)$ points. The value s may be chosen as $(t-m_p)/h$. When the tolerance is satisfied, the final "a" value is used as the shadow boundary time t_p. Use the current value of the partials at t_p for initial values and set $t_f=t_p$.

Enter the starting procedure and use the Taylor expansion in (4.1) to determine starting positions at the back values obtained with the reduced step h_1.

$$
\begin{gathered}
t_p+rh_1\\
t_p+rh_1-h_1\\
t_p+rh_1-2h_1\\
\cdots\\
t_p+rh1-(i-1)h_1\\
\text{or} \qquad t-sh \qquad \text{where}\\
s=(t-t_p-rh_1+jh_1)/h \qquad j=0,1,2,\ldots,(i-1)
\end{gathered}
\tag{7.1}
$$

Calculate accelerations at the above points and continue through section 4, page 11, STARTING PROCEDURE, by entering the accelerations into (4.9) with $t_f=t_p$. In this way $t_0=t_f+rh_1=t_f+t_d$ is consistent with (4.9). Since the value of ISHDOW is TRUE the correct functional value of *shape*, SHAPET, will be used to set the value CSHAPE. It is the CSHAPE value which is used during these calculations. This is because the *shape* function is changing while entering shadow and the difference table should reflect that. When exiting shadow the *shape* function is going to be constant and the difference table should reflect that. This will be accomplished by having ISHDOW set to FALSE causing CSHAPE to be set to $shape(t_p)$ during the starting procedure. It will remain at this value until the shadow region is entered again.

The *shape* function takes on values at each time line and the value at the current time line is called SHAPET. However, the value of the *shape* function that is used during the start procedure may differ from the value of SHAPET at the current time line. Thus, a value CSHAPE, which is generally equal to SHAPET, the value of the *shape* function at the current time line t, is used,

and this value will sometimes not be equal to SHAPET, as was indicated above in exiting a shadow region.

Continue through the starting procedure but do not generate the negative time lines which were necessary for interpolation at epoch.

After convergence in the starting procedure is reached, enter section 5, page 15, RUNNING PROCEDURE, with the new step h_1.

When shadow testing indicates that the shadow has been exited, ISHDOW will have been set to FALSE. A shadow restart time will be determined in the same manner as discussed for entering the shadow.

7.2 Exiting Shadow

Determine the shadow boundary time, t_p, by setting $a = t - rh_1 - h_1$, $b = t - rh_1$, where t_p is within this interval. Compute the midpoint, $m_p = (a+b)/2$. Redefine "a" and "b" in such a way that the new interval, $[a,b]$, will be $[a,m_p]$ or $[m_p,b]$. Choose the new "a" and "b" values such that the *shape* value at the new "a" is less(greater) than 1(0) and the shape value at the new "b" is 1(0). Continue this process until $b-a$ is less than a tolerance. Use (5.3) to calculate the various $x(m_p)$ points. The value s may be chosen as $(t-m_p)/h_1$. When the tolerance is satisfied, the final "a" value is used as the shadow boundary time t_p. Use the current value of the partials at t_p for initial values and set $t_f = t_p$. Enter section 4, page 11, STARTING PROCEDURE, with step h_0, $t_d = rh_0$, and $t_0 = t_f + t_d$. Since the satellite is now exiting the shadow region the *shape* function is going to be constant and the difference table should reflect that. This is accomplished by having the flag ISHDOW set to FALSE causing the value of CSHAPE to take on the constant value given by $shape(t_p)$. This value for shape will be used until the shadow boundary is encountered again. Continue as described in Entering Shadow, with the exception that the new integration step is now h_0.

7.3 Shadow Testing and the *Shape* Function

The shadow test is carried out after each integration step to determine if the satellite is passing from sunlight to darkness or from darkness to sunlight. The shadow test uses a function $shape(t)$ defined in the next section, to carry out this test. The value of $shape(t)$ at any time t is denoted, in the program, by SHAPET. However, the value used in the calculation of the solar radiation force, at time t, is CSHAPE which is not necessarily equal to $shape(t)$ =SHAPET. This is a consequence of the starting procedure. At certain restart times the radiation force scaling constant, CSHAPE, is held constant during start. This is done by having the shadow flag, ISHDOW, set to FALSE during that time. No change is made in the radiation force shadow scaling constant, CSHAPE, unless the shadow flag, ISHDOW, is set to TRUE. In this event the value of CSHAPE is set to the value of the *shape* function at time t, SHAPET.

The value of SHAPET is calculated at each time line t. Its value at any time is the ratio of the portion of the Sun's cross-sectional area not obscured from the view of the satellite to the total Sun cross-sectional area. Thus, when the satellite is in full sunlight, the value of SHAPET is

one, and when it is in full darkness, the value of SHAPET is zero. The value of SHAPET when the satellite is in shadow (prenumbra) is calculated as the ratio described above and the details of the calculation are given in the next section. Using the *shape* function, a test is carried out at each time line.

If ISHDOW is FALSE, then the satellite is either in full sunlight or full darkness and the shadow test is carried out by examining the value of the *shape* function at $t-(r+1)/h$ and $t-rh$. If, in going from $t-(r+1)h$ to $t-rh$ *shape* has changed from zero to a value other than zero, or from one to a value other than one, then the satellite is considered to be entering a shadow region and the value of ISHDOW is set to TRUE. In addition section 7, page 19, SHADOW RESTART, computes the shadow boundary time, t_p, in preparation for restarting the integration at a smaller time step h_1. During the force computations CSHAPE is set to SHAPET, as a result of checking ISHDOW. This dynamically changing value of the *shape* function is used throughout the shadow region, both in the starting procedure on entering shadow and in the running procedure while in the shadow.

If ISHDOW is TRUE, then the satellite is in the shadow (prenumbra) and the test is carried out by examining the value of the *shape* function at $t-(r+1)h$ and $t-rh$. If, in going from $t-(r+1)h$ to $t-rh$, *shape* has changed from a value other than zero to zero, or from a value other than one to one, then the satellite is considered to be exiting a shadow region and the value of ISHDOW is set to FALSE. In addition SHADOW RESTART computes the shadow boundary time, t_p, in preparation for restarting the integration at a larget step h_0. During the force computations CSHAPE is set to $shape(t_p)$. This constant value is used during the starting procedure on exiting shadow and in the subsequent running procedure.

7.4 *Shape* Computation

The *shape* computation is used to determine the functions numerical value. This value, as discussed earlier, represents the fraction of the Sun's rays which are not obscured from the satellite by the Earth and the Moon. When this value is not one the satellite is in some phase of eclipse. This could be an eclipse of the Sun by the Moon or an eclipse of the Sun by the Earth, relative to the satellite. When this occurs the integration process changes the integration step size to accommodate the force change at the shadow boundary and also calculates yaw information to write on the ephemeris output file for use in subsequent data processing. All shadow crossing times are also written to the output ephemeris file. Using Figure 1 below, and similar figures which could be constructed for the Moon-Sun and Earth-Moon pairs, fundamental quanties Θ_{es}, α_e, α_s , and Θ_{ms}, Θ_{em}, α_m can be used to set forth the computation of the *shape* function.

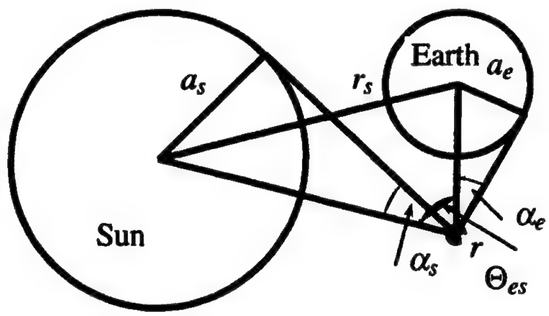


Figure 1: Satellite-Earth-Sun Shadow Diagram

$$
\begin{aligned}
Q_s &= ((r_s - r)^T (r_s - r) - a_s^2)^{1/2} \\
Q_e &= (r^T r - a_e^2)^{1/2} \\
Q_m &= ((r_m - r)^T (r_m - r) - a_m^2)^{1/2}
\end{aligned}
\tag{7.2}
$$

$$\cos(\Theta_{es}) = -r^T (r_s - r)/(|r||r_s - r|) \tag{7.3}$$

$$\cos(\Theta_{em}) = -r^T (r_m - r)/(|r||r_m - r|) \tag{7.4}$$

$$\cos(\Theta_{ms}) = (r_m - r)^T (r_s - r)/(|r_m - r||r_s - r|) \tag{7.5}$$

$$\cos(\alpha_e) = Q_e/|r| \tag{7.6}$$

$$\sin(\alpha_e) = a_e/|r| \tag{7.7}$$

$$\cos(\alpha_s) = Q_s/|r_s - r| \tag{7.8}$$

$$\sin(\alpha_s) = a_s/|r_s - r| \tag{7.9}$$

$$\cos(\alpha_m) = Q_m/|r_m - r| \tag{7.10}$$

$$\sin(\alpha_m) = a_m/|r_m - r| \tag{7.11}$$

Define intervals (d_{es}, d_{es}^1) , (d_{ms}, d_{ms}^1) and (d_{em}, d_{em}^1) depicted below.

$$
\begin{aligned}
d_{es} &= Q_e Q_s - a_e a_s & d_{es}^1 &= Q_e Q_s + a_e a_s \\
d_{ms} &= Q_m Q_s - a_m a_s & d_{ms}^1 &= Q_m Q_s + a_m a_s \\
d_{em} &= Q_e Q_m - a_e a_m & d_{em}^1 &= Q_e Q_m + a_e a_m
\end{aligned}
\tag{7.12}
$$

$$
\underline{\qquad}(\underset{d_{es}}{\underline{\qquad\qquad}})\underset{d_{es}^1}{\underline{\qquad\qquad}}(\underset{d_{ms}}{\underline{\qquad\qquad}})\underset{d_{ms}^1}{\underline{\qquad\qquad}}(\underset{d_{em}}{\underline{\qquad\qquad}})\underset{d_{em}^1}{\underline{\qquad\qquad}}
$$

SHAPET has a value between zero and one. The computation of this value is carried out by defining areas associated with the angles displayed in the Satellite-Earth-Sun diagram above. The areas are proportional to the angles and are given by

$$A_s = \pi\alpha_s^2 \qquad A_e = \pi\alpha_e^2 \qquad A_m = \pi\alpha_m^2 \tag{7.13}$$

to represent Sun, Earth and Moon area cross-sections. Areas, A_{es} and A_{ms} represent how much of the Sun is obscured by the Earth and Moon. A_{em} represents how much of the Moon is obscured by the Earth.

Define the areas A_{ij}, where i and j represent Earth(e), Sun(s), and Moon(m), by

$$\begin{aligned}
&\text{If} \quad \theta_{ij} \geq \alpha_i + \alpha_j \quad \text{or, equivalently,}\\
&\quad (r_i - r)^T (r_j - r) \leq d_{ij}\\
&\text{Then,} \quad A_{ij} = 0
\end{aligned} \tag{7.14}$$

$$\begin{aligned}
&\text{If} \quad \theta_{ij} \leq | \alpha_i - \alpha_j | \quad \text{or, equivalently,}\\
&\quad (r_i - r)^T (r_j - r) \geq d_{ij}^1\\
&\text{Then,} \quad A_{ij} = min(A_i, A_j)
\end{aligned} \tag{7.15}$$

$$\begin{aligned}
&\text{If} \quad | \alpha_i - \alpha_j | < \theta_{ij} < \alpha_i + \alpha_j \quad \text{or, equivalently,}\\
&\quad d_{ij} < (r_i - r)^T (r_j - r) < d_{ij}^1\\
&\text{Then,} \quad A_{ij} = \alpha_i^2 \cos^{-1}((\theta_{ij}^2 + \alpha_i^2 - \alpha_j^2)/(2\theta_{ij}\alpha_i))\\
&\quad + \alpha_j^2 \cos^{-1}((\theta_{ij}^2 + \alpha_j^2 - \alpha_i^2)/(2\theta_{ij}\alpha_j))\\
&\quad - (1/2)(4\alpha_i^2\alpha_j^2 - (\theta_{ij}^2 - \alpha_i^2 - \alpha_j^2)^2)^{1/2}\\
&\quad \text{Where,} \quad r_e = 0 \quad \text{(distance from Earth center to Earth center)}
\end{aligned} \tag{7.16}$$

In general, the only test that should be carried out is the first test for $A_{es} = 0$ and $A_{ms} = 0$, as this is the situation most of the time and, if this condition holds, then there is no need for further calculation. Cases 1-4 below, displayed in Figures 2-8, cover the calculation of the *shape* function.

Case 1, Figures 3-5, satisfies the following special conditions of the above formula.

$$\begin{aligned}
-r^T (r_s - r) &\leq d_{es} & A_{es} &= 0\\
(r_m - r)^T (r_s - r) &\leq d_{ms} & A_{ms} &= 0\\
-r^T (r_m - r) &\leq d_{em} & A_{em} &= 0
\end{aligned} \tag{7.17}$$

$$\begin{aligned}
-r^T (r_s - r) &\geq d_{es}^1 & A_{es} &= min(A_e, A_s) = A_s\\
(r_m - r)^T (r_s - r) &\geq d_{ms}^1 & A_{ms} &= min(A_m, A_s)\\
-r^T (r_m - r) &\geq d_{em}^1 & A_{em} &= min(A_e, A_m) = A_m
\end{aligned} \tag{7.18}$$

The calculation of the *shape* function, for this special case, is covered by Case 1, i.e., Cases 1A - 1F. If any of the above special cases do not hold then the general formula above for A_{ij} must be

used. In that event, all such cases satisfy the conditions

$$A_{es}, A_{ms}, A_{em} > 0 \qquad A_{es} < A_s$$
$$A_{ms} < min(A_m, A_s) \quad A_{em} < A_m \tag{7.19}$$

and are covered in Cases 2A - 2D, 3A - 3B, and 4A. Figure 2 and Figures 6-8 cover these cases. The cases are broken down by having the Earth-Sun circle intersection points, R and R', satisfy the condition that they are both external to the Moon circle, both internal to the Moon circle, or one internal and one external to the Moon circle, respectively. Critical values are as follows and are used to calculate the shape function for the above cases. The calculations appear in the figures below.

$(0,0)$	Sun center	
(xm, ym)	Moon center	
$(\theta_{es}, 0)$	Earth center	
$(x_R, y_R), (x_{R'}, y_{R'})$	Earth-Sun intersections	(7.20)
$(x_S, y_S), (x_{S'}, y_{S'})$	Earth-Moon intersections	
$(x_T, y_T), (x_{T'}, y_{T'})$	Sun-Moon intersections	
A_{esm}	convex area of RST in Case 4A	

Moon-Center

$$x_m^2 + y_m^2 = \theta_{ms}^2$$
$$(\theta_{es} - x_m)^2 + y_m^2 = \theta_{em}^2$$
yields
$$x_m = (\theta_{es}^2 + \theta_{ms}^2 - \theta_{em}^2)/(2\theta_{es})$$
$$y_m = (\theta_{ms}^2 - x_m^2)^{1/2}$$

Earth-Sun Intersection

$$(x_R - \theta_{es})^2 + y_R^2 = \alpha_e^2$$
$$x_R^2 + y_R^2 = \alpha_s^2$$
yields
$$x_R = (\theta_{es}^2 + \alpha_s^2 - \alpha_e^2)/(2\theta_{es})$$
$$y_R = (\alpha_s^2 - x_R^2)^{1/2}$$

Earth-Moon Intersection

$$(x_S-\theta_{es})^2+y_S^2=\alpha_e^2$$
$$(x_S-x_m)^2+(y_S-y_m)^2=\alpha_m^2$$
yields
$$x_S=\theta_{es}+z_s$$
$$y_S=(\alpha_e^2-z_s^2)^{1/2}$$
$$z_s=(\gamma/(2\theta_{em}^2))[x_m-\theta_{es}$$
$$-((4\alpha_e^2\theta_{em}^2/\gamma^2)-1)^{1/2}]$$
$$\gamma=\alpha_e^2-\alpha_m^2+\theta_{em}^2$$

These values are consistant with Case 4A, but not with the other Cases. Cases 1, 2, and 3 have S and S' interchanges from how they are labeled in Case 4A. These equations take

$$x_S=min(x_S,x_{S'})$$
$$y_S\geq 0$$

Sun-Moon Intersection

$$x_T^2+y_T^2=\alpha_s^2$$
$$(x_T-x_m)^2+(y_T-y_m)^2=\alpha_m^2$$
yields
$$x_T=(\beta/(2\theta_{ms}^2))[x_m+y_m((4\alpha_s^2\theta_{ms}^2/\beta^2)-1)^{1/2}]$$
$$y_T=(\alpha_s^2-x_T^2)^{1/2}$$
$$\beta=\alpha_s^2-\alpha_m^2+\theta_{ms}^2$$

The signs have been chosen so that

$$x_T=max(x_T,x_{T'})$$
$$y_T\geq 0$$

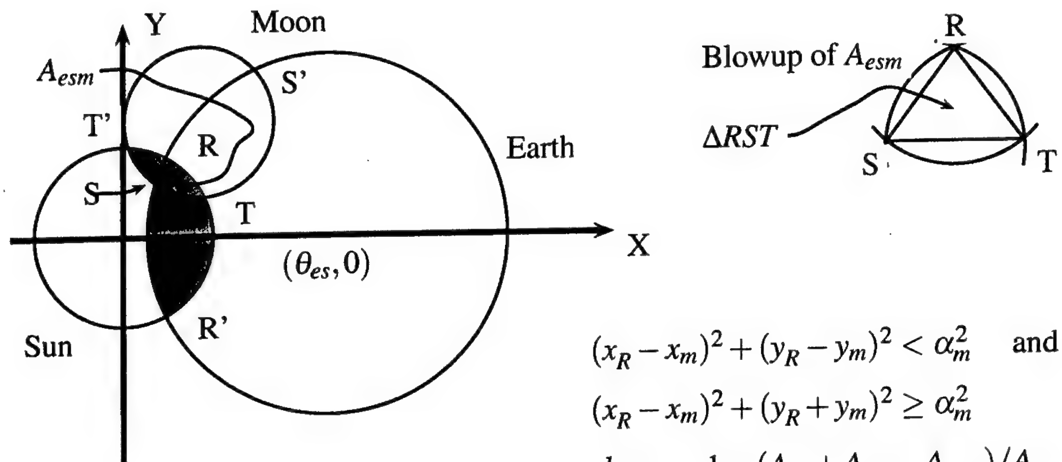
Area of Region $RST=A_{esm}$

Form arcs RS, RT, ST with corresponding lengths L_{RS}, L_{RT}, L_{ST}. The regions above the arcs on the Earth circle, Sun circle, and Moon circle, respectively, will have areas

$$B_{RS}=\alpha_e^2\cos^{-1}(D_{RS}/\alpha_e)-D_{RS}L_{RS}/2$$
$$D_{RS}=(\alpha_e^2-(L_{RS}/2)^2)^{1/2}$$
$$B_{RT}=\alpha_s^2\cos^{-1}(D_{RT}/\alpha_s)-D_{RT}L_{RT}/2$$
$$D_{RT}=(\alpha_s^2-(L_{RT}/2)^2)^{1/2}$$
$$B_{ST}=\alpha_m^2\cos^{-1}(D_{ST}/\alpha_m)-D_{ST}L_{ST}/2$$
$$D_{ST}=(\alpha_m^2-(L_{ST}/2)^2)^{1/2}$$

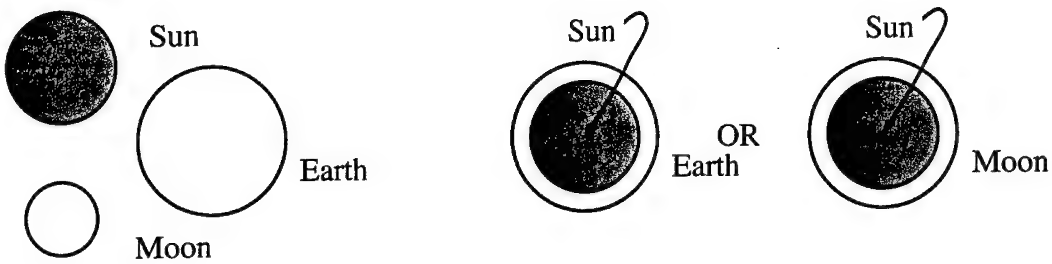
The sum of these areas, B, together with the area of triangle RST, ΔRST, will be the total area, A_{esm}, of the convex region RST.

$$
\begin{aligned}
B &= B_{RS} + B_{RT} + B_{ST} \\
\Delta RST &= [C(C - L_{RS})(C - L_{RT})(C - L_{ST})]^{1/2} \\
C &= (L_{RS} + L_{RT} + L_{ST})/2 \\
A_{esm} &= B + \Delta RST
\end{aligned}
$$



$$
\begin{aligned}
&(x_R - x_m)^2 + (y_R - y_m)^2 < \alpha_m^2 \quad \text{and} \\
&(x_R - x_m)^2 + (y_R + y_m)^2 \geq \alpha_m^2 \\
&shape = 1 - (A_{es} + A_{ms} - A_{esm})/A_s
\end{aligned}
$$

Figure 2: Satellite-Earth-Sun Shadow Case 4A



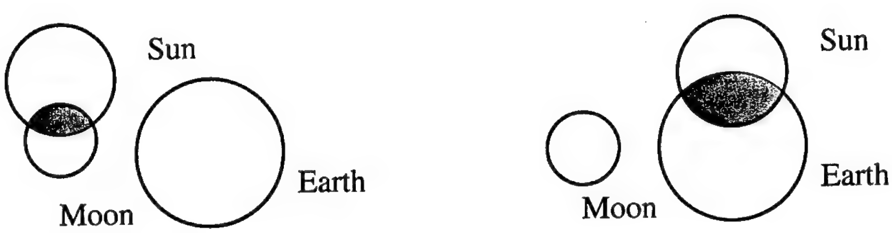
$A_{es} = A_{ms} = 0 \, shape = 1$

$A_{es} = A_s \quad or \quad A_{ms} = A_s \, shape = 0$

Case 1A

Case 1B

Figure 3: Satellite-Earth-Sun Shadow Case 1A and Case 1B



$A_{es} = 0 \quad 0 < A_{ms} < A_s$

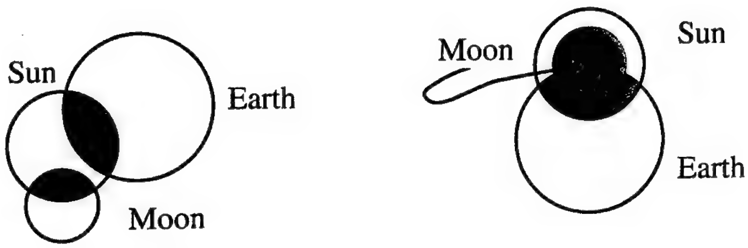
$shape = 1 - A_{ms}/A_s$

$A_{ms} = 0 \quad or \quad A_{em} = Am \quad and$
$0 < A_{es} < A_s \quad shape = 1 - A_{es}/A_s$

Case 1C

Case 1D

Figure 4: Satellite-Earth-Sun Shadow Case 1C and Case 1D



$A_{em} = 0 \quad 0 < A_{es} < A_s \quad 0 < A_{ms} < A_s$

$shape = 1 - (A_{es} + A_{ms})/A_s$

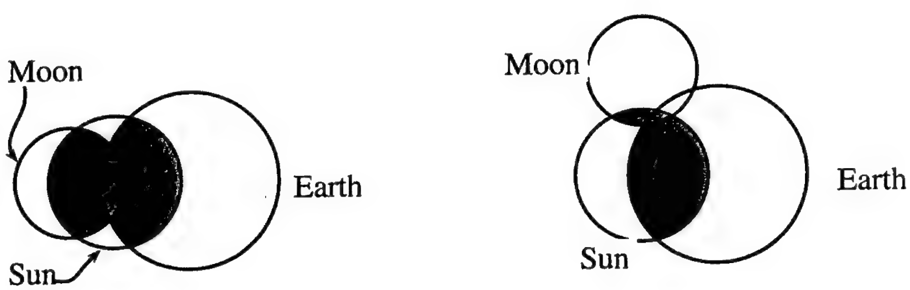
$A_{ms} = A_m \quad A_{em} \neq 0 \quad and 0 < A_{es} < A_s$

$shape = 1 - (A_{es} + A_{ms} - A_{em})/A_s$

Case 1E

Case 1F

Figure 5: Satellite-Earth-Sun Shadow Case 1E and Case 1F



$x_m \leq x_R$ and $x_S^2 + y_S^2 < \alpha_s^2$

$shape = 1 - (A_{es} + A_{ms} - A_{em})/As$

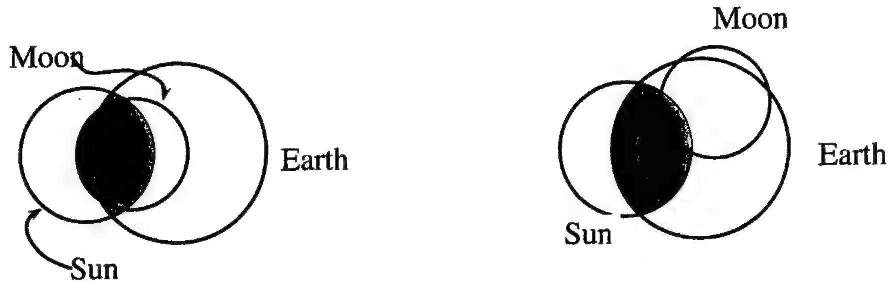
$x_m \leq x_R$ and $x_S^2 + y_S^2 \geq \alpha_s^2$

$shape = 1 - (A_{es} + A_{ms})/As$

Case 2A

Case 2B

Figure 6: Satellite-Earth-Sun Shadow Case 2A and Case 2B



$x_m > x_R$ and $x_S^2 + y_S^2 < \alpha_s^2$

$shape = 1 - (A_{es} + A_m - A_{em})/As$

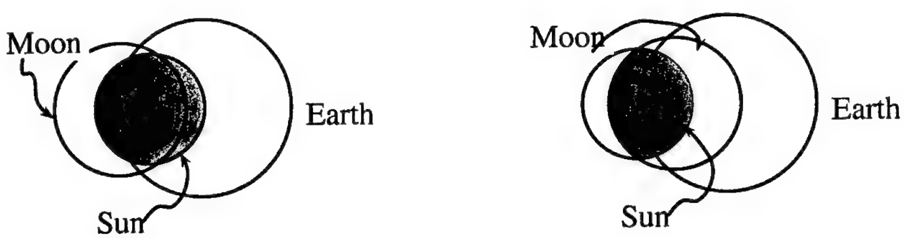
Case 2C

$x_m > x_R$ and $x_S^2 + y_S^2 \geq \alpha_s^2$

$shape = 1 - A_{es}/As$

Case 2D

Figure 7: Satellite-Earth-Sun Shadow Case 2C and Case 2D



$(x_R - x_m)^2 + (y_R - y_m)^2 < \alpha_m^2$ and
$(x_R - x_m)^2 + (y_R + y_m)^2 < \alpha_m^2$
$(\alpha_s - x_m)^2 + y_m^2 > \alpha_m^2$
$shape = 0$
Case 3A

$(x_R - x_m)^2 + (y_R - y_m)^2 < \alpha_m^2$ and
$(x_R - x_m)^2 + (y_R + y_m)^2 < \alpha_m^2$
$(\alpha_s - x_m)^2 + y_m^2 \leq \alpha_m^2$
$shape = 1 - A_{ms}/A_s$
Case 3B

Figure 8: Satellite-Earth-Sun Shadow Case 3A and Case 3B

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8 INTEGRATION RESTART

Each time the word thrust appears below, the words drag, momentum dump, or radiation shadow boundary may be used in its place. Each has its own integration interval.

When, t, the first point greater than or equal to the start of thrust, t_s, is reached, such that $t - rh$ is also greater than or equal to t_s then, after converging at t with no thrust, compute $x(t_s)$ using (5.3), with $s = (t - t_s)/h$. Enter section 4, page 11, with the time of the fixed point $t_f = t_s$, fixed point $x(t_f) = x(t_s)$, integration step $h = h_2$, time of the difference table, measured from t_f, being given by $t_d = rh_2$, and the time of the difference table (zero line of the difference table) given by $t_0 = t_f + t_d$. Use the current value of the partials at t_s for initial partial values. Enter section 5, page 15, and continue integrating with step h.

When the first point, t, is reached, which is greater than or equal to the end of the thrust, t_e, and such that the previous point, $t - rh_2$, is also greater than or equal to t_e, converge the solution at t, with thrust on, and calculate $x(t_e)$ using (5.3), with $s = (t - t_e)/h_2$.

Enter section 4 with the time of the fixed point, $t_f = t_e$, fixed point $x(t_f) = x(t_e)$, integration step $h = h_0$, time of the difference table measured from t_f, being given by $t_d = rh_0$, and the time of the difference table given by $t_0 = t_f + t_d$. Use the current value of the partials at t_e for initial partial values.

Enter section 5, page 15, and continue integrating with step h.

8.1 Restart Sorting

In the event more than one restart occurs during the interval $(t - (r+1)h,\quad t - rh]$, it is necessary to sort the restart times into ascending order and use the earliest restart time as the actual restart time. Should more than one restart occur at the same earliest restart time, then the integration step size must be taken as the smallest of the restart steps associated with the simultaneous earliest restart times.

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9 BACKDATE

If the backdate integration option is executed then the initialization must be carried out in the following, slightly different, manner to that of standard forward integration.

The backdate integration process is initialized by changing the initial conditions $(x(0),\ \dot{x}(0))$ to $(x(0),\ -\dot{x}(0))$, changing the drag value, C_d to $-C_d$, negating the T_2 and T_3 thrust coefficients if the (RVC) or (RAC) thrust frame, in 11.5.1, page 57, is being used, and negating the Earth's angular velocity ω_d. In addition, the absolute (year, day, second) time is stepped backward from the initial time, via $T = T_0 - t$, when obtaining values for the positions of planets from the Sun-Moon ephemeris file. Additional initialization may be required if partial derivatives are to be referenced to the original initial conditions $(x(0),\dot{x}(0))$ or to their associated osculating orbital elements. Recall the osculating orbital elements from (2.2).

$$\begin{aligned} e_1 &= a & e_4 &= i \\ e_2 &= e\sin(\omega) & e_5 &= l+\omega \\ e_3 &= e\cos(\omega) & e_6 &= \Omega \end{aligned}$$

For each $(x,\ \dot{x})$ there is an associated $(x,\ -\dot{x})$. Associated to the position, velocity $(x,\ \dot{x})$ there is a set of orbital elements $e(x,\ \dot{x}) = (e_1,\ e_2,\ \ldots,\ e_6)$. Associated with $(x,\ -\dot{x})$ there is the corresponding set of hatted elements, $e(x,\ -\dot{x}) = (\hat{e}_1,\ \hat{e}_2,\ \ldots,\ \hat{e}_6)$. Since the orbit associated with $(x,\ -\dot{x})$ is geometrically the same but moving in the opposite direction to that of the one associated with $(x,\ \dot{x})$, the ascending node of the former will be 180 degrees from that of the latter. This implies the relationship among the classical elements

$$\begin{aligned} \hat{a} &= a & \hat{i} &= 180^o - i \\ \hat{e} &= e & \hat{l} &= 360^o - l \\ \hat{\omega} &= -180^o - \omega & \hat{\Omega} &= 180^o + \Omega \end{aligned} \tag{9.1}$$

which in turn implies the osculating element relationship

$$\begin{aligned} \hat{e}_1 &= e_1 & \hat{e}_4 &= 180^o - e_4 \\ \hat{e}_2 &= e_2 & \hat{e}_5 &= 180^o - e_5 \\ \hat{e}_3 &= -e_3 & \hat{e}_6 &= 180^o + e_6 \end{aligned} \tag{9.2}$$

Utilizing the above, the value of $\partial\hat{e}/\partial e$ is given by

$$\partial\hat{e}/\partial e = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{9.3}$$

Setting $(y(t),\ \dot{y}(t)) = (x(t),\ -\dot{x}(t))$ gives the relationship between the y variables of backward integration and the x variables to be saved.

Given a set of epoch conditions $(x(0),\ \dot{x}(0))$ there is a corresponding set of orbital elements $e(x(0),\ \dot{x}(0))$, a set of epoch conditions for backdating $(y(0),\ \dot{y}(0)) = (x(0),\ -\dot{x}(0))$, and the corresponding hatted set of orbital elements $\hat{e} = e(x(0),\ -\dot{x}(0))$. It is desired to compute

$$\partial(y(t),\dot{y}(t))/\partial e(0) = \partial(x(t),-\dot{x}(t))/\partial e(0) \tag{9.4}$$

and to save $\partial(x(t),\dot{x}(t))/\partial e(0)$ at each time $T_0 - t$.

Desiring to compute $\partial(y(t),\dot{y}(t))/\partial e(0)$ and utilizing

$$\begin{aligned}\partial(y(t),\dot{y}(t))/\partial e(0) &= \partial(y(t),\dot{y}(t))/\partial\hat{e}(0)\cdot\partial\hat{e}(0)/\partial e(0)\\ &= \partial(y(t),\dot{y}(t))/\partial\hat{e}(0)\cdot\partial\hat{e}/\partial e\end{aligned} \tag{9.5}$$

the partial derivatives for backdating in element mode should be initialized as

$$\partial(y(0),\dot{y}(0))/\partial e(0) = \partial(y(0),\dot{y}(0))/\partial\hat{e}(0)\cdot\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&-1&0&0&0\\0&0&0&-1&0&0\\0&0&0&0&-1&0\\0&0&0&0&0&1\end{bmatrix} \tag{9.6}$$

The term $\partial(y(0),\dot{y}(0))/\partial\hat{e}(0)$ represents the partial derivatives of rectangular coordinates with respect to osculating orbital elements evaluated at the epoch set $(x(0),\ -\dot{x}(0))$.

Desiring to compute $\partial(y(t),\dot{y}(t))/\partial(x(0),\dot{x}(0))$ in coordinate mode, implies that the initial conditions should be given by

$$\partial(y(0),\dot{y}(0))/\partial(x(0),\dot{x}(0) = \begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&-1&0&0\\0&0&0&0&-1&0\\0&0&0&0&0&-1\end{bmatrix} \tag{9.7}$$

Thus, the initial conditions for backdate are given by

$$\begin{bmatrix}y(0)\\ \dot{y}(0)\\ \partial(y(0),\dot{y}(0))/\partial e(0)\end{bmatrix} \tag{9.8}$$

for element mode, and

$$\begin{bmatrix} y(0) \\ \dot{y}(0) \\ \partial(y(0),\dot{y}(0))/\partial(x(0),\dot{x}(0))) \end{bmatrix} \tag{9.9}$$

for coordinate mode.

During backward integration, output values should be a subset of the following in inertial, Earth-fixed, or both frames.

Element mode

$$\begin{bmatrix} y(t) \\ -\dot{y}(t) \\ \partial y(t)/\partial e(0) \\ -\partial \dot{y}(t)/\partial e(0 \end{bmatrix} \tag{9.10}$$

Coordinate mode

$$\begin{bmatrix} y(t) \\ -\dot{y}(t) \\ \partial y(t)/\partial(x(0),\dot{x}(0)) \\ -\partial \dot{y}(t)/\partial(x(0),\dot{x}(0)) \end{bmatrix} \tag{9.11}$$

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10 OUTPUT

- Polar Motion Information (Types 0 1 2 5 6)
- Earth-fixed Position and Velocity (Center of Mass) (Types 0 1 2 5 6)
- Earth-fixed Position and Velocity (Antenna) (Selected Satellites)
- Inertial Position (Types 0 1 2 5 6)
- Inertial Velocity (Type 5)
- Partial Derivatives of the Inertial Position with respect to Initial Position and Velocity or Initial Osculating Orbital Elements (Types 0 1 5 6)
- Body-fixed Radiation Pressure Acceleration Values (Types 0 1 5 6)
- Body-fixed to Inertial Transformation Matrix (Types 0 1 5 6)
- Radiation Pressure Eclipse Indicator, i.e., Shape value (Types 0 1 5 6)
- Drag Acceleration Values (Type 6)
- Satellite Longitude Latitude and Height (Type 6)
- Sun Position (Type 6)
- Radiation Shadow Boundary Crossing Times (Types 0 1 5 6)
- Yaw Attitude Information (Selected Satellites) [Bar-Sever-5]
- Other Information (See Ephemeris File Description, section 14.10, page 138)

The output ephemeris contains all of the above information. The Polar Motion information is in the header of the output trajectory and the remaining values are in the tabular records. Tabular records occur at equal intervals of the input write interval. How much of the above information is contained in the tabular record depends on the input option of TRAJECTORY TYPE, indicated above by integers (0 - 6) and provided in section 14.2, page 121.

Trajectory TYPE of 3 and 4 represent initial condition update or backdate to obtain values at a different epoch. Output is provided in the form of new initial conditions at the desired epoch. Results will be on the initial conditions file at 14.5, page 130, as well as printed on the program unit IOUTFL/6, which is usually the display. No ephemeris output file is provided for these types.

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11 FORCES

11.1 Earth Gravity

The gravity potential and associated acceleration can be expressed as

$$V = \sum_{n=0}^{N}\sum_{m=0}^{n}\left[C_{nm}\frac{\mu}{|r|}\left(\frac{a_e}{|r|}\right)^n P_{nm}(g'/|r|)\cos(m\lambda) + S_{nm}\frac{\mu}{|r|}\left(\frac{a_e}{|r|}\right)^n P_{nm}(g'/|r|)\sin(m\lambda)\right] \tag{11.1}$$

$$V = \sum_{n=0}^{N}\sum_{m=0}^{n}\left[C_{nm}U_n^m + S_{nm}V_n^m\right] \tag{11.2}$$

$$A_{earth} = E^T a_e \nabla V = E^T \sum_{n=0}^{N}\sum_{m=0}^{n}\left[C_{nm}a_e \nabla U_n^m + S_{nm}a_e \nabla V_n^m\right] \tag{11.3}$$

where (e', f', g') are the Earth-fixed (see Remark below) satellite coordinates, μ is the gravitational constant, λ is Greenwich longitude, $g'/|r| = \cos(\theta)$ where θ is the co-latitude, a_e is the Earth semi-major axis, $P_n(x) = \frac{1}{2^n n!}\frac{d^n(x^2-1)^n}{dx^n}$ is the Legendre polynomial, and $P_{nm}(x) = (1-x^2)^{m/2}\frac{d^m P_n(x)}{dx^m}$ is the associated Legendre polynomial. $E = BCD/a_e$ is the inertial-to-Earth-fixed transformation divided by the Earth's semi-major axis. The convention is $U_n^m = U_{column}^{row}$ and $C_{nm} = C_{columm\ row}$ (unnormalized). n is degree and m is order for the C_{nm} S_{nm} geopotential coefficients.

The goal in this gravity evaluation is to express the acceleration in the same manner as the potential. In order to do this it will be useful to express the gradient of the U_n^m, V_n^m in terms of the U_n^m, V_n^m, as in [Dewitt].

$$a_e \nabla U_n^m = \begin{bmatrix} (1/2)(n-m+1)(n-m+2)U_{n+1}^{m-1} - (1/2)U_{n+1}^{m+1} \\ -(1/2)(n-m+1)(n-m+2)V_{n+1}^{m-1} - (1/2)V_{n+1}^{m+1} \\ -(n-m+1)U_{n+1}^{m} \end{bmatrix} \tag{11.4}$$

$$a_e \nabla V_n^m = \begin{bmatrix} (1/2)(n-m+1)(n-m+2)V_{n+1}^{m-1} - (1/2)V_{n+1}^{m+1} \\ (1/2)(n-m+1)(n-m+2)U_{n+1}^{m-1} + (1/2)U_{n+1}^{m+1} \\ -(n-m+1)V_{n+1}^{m} \end{bmatrix} \tag{11.5}$$

Using the fact that each gradient ∇U_n^m, ∇V_n^m term is a linear combination of U_n^m, V_n^m terms, we can express the acceleration as

$$E^T \nabla V = E^T \sum_{n=1}^{N+1}\sum_{m=-1}^{n}\begin{bmatrix} C_{nm}^1 & S_{nm}^1 \\ C_{nm}^2 & S_{nm}^2 \\ C_{nm}^3 & S_{nm}^3 \end{bmatrix}\begin{bmatrix} U_n^m \\ V_n^m \end{bmatrix} \tag{11.6}$$

Thus the acceleration can be expressed in the same manner as the potential. The C^i_{nm} S^i_{nm} with $-1 \leq m \leq n$ and $1 \leq n \leq N+1$ are given by

$$C^1_{nm} = (1/2)(n-m-1)(n-m)C_{n-1,m+1} - (1/2)C_{n-1,m-1} \tag{11.7}$$
$$C^2_{nm} = (1/2)(n-m-1)(n-m)S_{n-1,m+1} + (1/2)S_{n-1,m-1} \tag{11.8}$$
$$C^3_{nm} = -(n-m)C_{n-1,m} \tag{11.9}$$
$$S^1_{nm} = (1/2)(n-m-1)(n-m)S_{n-1,m+1} - (1/2)S_{n-1,m-1} \tag{11.10}$$
$$S^2_{nm} = -(1/2)(n-m-1)(n-m)C_{n-1,m+1} - (1/2)C_{n-1,m-1} \tag{11.11}$$
$$S^3_{nm} = -(n-m)S_{n-1,m} \tag{11.12}$$

Applying this procedure to the double gradient gives $E^T \nabla^2 VE =$

$$E^T \sum_{n=2}^{N+2} \sum_{m=-2}^{n} \begin{bmatrix} C^{11}_{nm} & S^{11}_{nm} & C^{12}_{nm} & S^{12}_{nm} & C^{13}_{nm} & S^{13}_{nm} \\ C^{21}_{nm} & S^{21}_{nm} & C^{22}_{nm} & S^{22}_{nm} & C^{23}_{nm} & S^{23}_{nm} \\ C^{31}_{nm} & S^{31}_{nm} & C^{32}_{nm} & S^{32}_{nm} & C^{33}_{nm} & S^{33}_{nm} \end{bmatrix} \begin{bmatrix} U^m_n & 0 & 0 \\ V^m_n & 0 & 0 \\ 0 & U^m_n & 0 \\ 0 & V^m_n & 0 \\ 0 & 0 & U^m_n \\ 0 & 0 & V^m_n \end{bmatrix} E \tag{11.13}$$

Thus the double gradient can be expressed in the same manner as the potential. The C^{ij}_{nm} S^{ij}_{nm} with $-2 \leq m \leq n$ and $2 \leq n \leq N+2$ are given by

$$C^{i1}_{nm} = (1/2)(n-m-1)(n-m)C^i_{n-1,m+1} - (1/2)C^i_{n-1,m-1} \tag{11.14}$$
$$C^{i2}_{nm} = (1/2)(n-m-1)(n-m)S^i_{n-1,m+1} + (1/2)S^i_{n-1,m-1} \tag{11.15}$$
$$C^{i3}_{nm} = -(n-m)C^i_{n-1,m} \tag{11.16}$$
$$S^{i1}_{nm} = (1/2)(n-m-1)(n-m)S^i_{n-1,m+1} - (1/2)S^i_{n-1,m-1} \tag{11.17}$$
$$S^{i2}_{nm} = -(1/2)(n-m-1)(n-m)C^i_{n-1,m+1} - (1/2)C^i_{n-1,m-1} \tag{11.18}$$
$$S^{i3}_{nm} = -(n-m)S^i_{n-1,m} \tag{11.19}$$

An examination of the C^i_{nm} S^i_{nm} C^{ij}_{nm} S^{ij}_{nm} using the fact that the C_{nm} S_{nm} are zero for $n < 0$, $n > N$, or $m > n$ shows that the following expressions are valid and can be used to compute all the C^i_{nm} S^i_{nm} C^{ij}_{nm} S^{ij}_{nm} terms. When computing the C^i_{nm} S^i_{nm} terms as a function of the C_{nm} S_{nm} simply surpress the "i" index on both sides of the equation. In this way a uniform formula can be given to compute all the various C S values.

These values can now be used to restate the expressions for the gradient and double gradient of the gravity potential. The C^{ij}_{nm} S^{ij}_{nm} with $-2 \leq m \leq n$ and $0 \leq n \leq N+2$ are given by

$$C^{i1}_{nm} = (1/2)(n-m-1)(n-m)C^{i}_{n-1,m+1} - (1/2)C^{i}_{n-1,m-1} \tag{11.20}$$
$$C^{i2}_{nm} = (1/2)(n-m-1)(n-m)S^{i}_{n-1,m+1} + (1/2)S^{i}_{n-1,m-1} \tag{11.21}$$
$$C^{i3}_{nm} = -(n-m)C^{i}_{n-1,m} \tag{11.22}$$
$$S^{i1}_{nm} = (1/2)(n-m-1)(n-m)S^{i}_{n-1,m+1} - (1/2)S^{i}_{n-1,m-1} \tag{11.23}$$
$$S^{i2}_{nm} = -(1/2)(n-m-1)(n-m)C^{i}_{n-1,m+1} - (1/2)C^{i}_{n-1,m-1} \tag{11.24}$$
$$S^{i3}_{nm} = -(n-m)S^{i}_{n-1,m} \tag{11.25}$$

We can now express the potential and the acceleration as

$$V = \sum_{n=0}^{N+2} \sum_{m=-2}^{n} \begin{bmatrix} C_{nm} & S_{nm} \end{bmatrix} \begin{bmatrix} U^m_n \\ V^m_n \end{bmatrix} \tag{11.26}$$

$$E^T \nabla V = E^T \sum_{n=0}^{N+2} \sum_{m=-2}^{n} \begin{bmatrix} C^1_{nm} & S^1_{nm} \\ C^2_{nm} & S^2_{nm} \\ C^3_{nm} & S^3_{nm} \end{bmatrix} \begin{bmatrix} U^m_n \\ V^m_n \end{bmatrix} \tag{11.27}$$

and the gradient of the acceleration as $\partial A_{earth}/\partial r = E^T \nabla^2 V E =$

$$E^T \sum_{n=0}^{N+2} \sum_{m=-2}^{n} \begin{bmatrix} C^{11}_{nm} & S^{11}_{nm} & C^{12}_{nm} & S^{12}_{nm} & C^{13}_{nm} & S^{13}_{nm} \\ C^{21}_{nm} & S^{21}_{nm} & C^{22}_{nm} & S^{22}_{nm} & C^{23}_{nm} & S^{23}_{nm} \\ C^{31}_{nm} & S^{31}_{nm} & C^{32}_{nm} & S^{32}_{nm} & C^{33}_{nm} & S^{33}_{nm} \end{bmatrix} \begin{bmatrix} U^m_n & 0 & 0 \\ V^m_n & 0 & 0 \\ 0 & U^m_n & 0 \\ 0 & V^m_n & 0 \\ 0 & 0 & U^m_n \\ 0 & 0 & V^m_n \end{bmatrix} E \tag{11.28}$$

The U^m_n V^m_n arrays are triangular arrays which are generated via recursion relations. The recursion relations are given by horizontal and diagonal stepping procedures, where $U^m_n = V^m_n = 0$ when $m > n$. Recall the convention $U^m_n = U^{row}_{column}$ and $C_{nm} = C_{column\ row}$.

Horizontal Stepping where $m \leq n$

$$U^m_{n+1} = (\rho/(n-m+1)) \left[(g'/|r|)(2n+1)U^m_n - (n+m)\rho U^m_{n-1} \right] \tag{11.29}$$

$$V^m_{n+1} = (\rho/(n-m+1)) \left[(g'/|r|)(2n+1)V^m_n - (n+m)\rho V^m_{n-1} \right] \tag{11.30}$$

and Diagonal Stepping where $m = n$

$$U^{n+1}_{n+1} = (2n+1)\rho \left[U^n_n(e'/|r|) - V^n_n(f'/|r|) \right] \tag{11.31}$$

$$V^{n+1}_{n+1} = (2n+1)\rho \left[V^n_n(e'/|r|) + U^n_n(f'/|r|) \right] \tag{11.32}$$

Negative U and V values are computed from

$$U_n^{-m} = (-1)^m((n-m)!/(n+m)!)U_n^m \tag{11.33}$$

$$V_n^{-m} = (-1)^{m+1}((n-m)!/(n+m)!)V_n^m \tag{11.34}$$

U and V values are initialized with

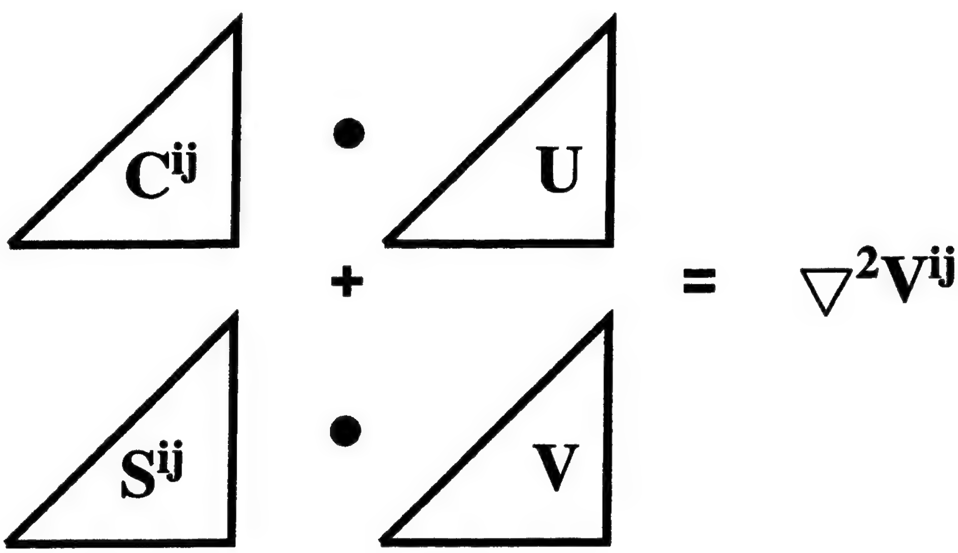
$$U_0^0 = \mu/|r| \qquad U_1^0 = \mu a_e g'/|r|^3 \tag{11.35}$$

$$V_0^0 = 0 \qquad V_1^0 = 0 \tag{11.36}$$

$$\rho = a_e/|r| \qquad |r| = (e'^2 + f'^2 + g'^2)^{1/2} \tag{11.37}$$

Figure 9 shows the computation of $\nabla^2 V^{ij}$, from equation (11.28).

$$\nabla^2 V^{ij} = \sum_{n=0}^{N+2} \sum_{m=-2}^{n} (C_{nm}^{ij} U_n^m + S_{nm}^{ij} V_n^m) \tag{11.38}$$



Dot Corresponding Columns of Upper Pair

Dot Corresponding Columns of Lower Pair

Add All Results

Figure 9: Pictorial Representation of Gravity Calculation

All of equation (11.28) can be expressed by

$$E^T \nabla^2 V E = E^T \begin{bmatrix} \nabla^2 V^{11} & \nabla^2 V^{12} & \nabla^2 V^{13} \\ \nabla^2 V^{21} & \nabla^2 V^{22} & \nabla^2 V^{23} \\ \nabla^2 V^{31} & \nabla^2 V^{32} & \nabla^2 V^{33} \end{bmatrix} E \tag{11.39}$$

Similarly equation (11.27) for the acceleration can be expressed by

$$\nabla V^i = \sum_{n=0}^{N+2} \sum_{m=-2}^{n} (C^i_{nm} U^m_n + S^i_{nm} V^m_n) \tag{11.40}$$

and all of equation (11.27) can be expressed by

$$E^T \nabla V = E^T \begin{bmatrix} \nabla V^1 \\ \nabla V^2 \\ \nabla V^3 \end{bmatrix} \tag{11.41}$$

In each case the computation involves calculating the dot product of two pairs of triangular matrices and adding the results. One pair consists of the C and U values and the other consists of the S and V values. The dot product of the two matrices is the sum of the products of all values with corresponding indices. This is the same as taking the dot product of corresponding columns of C with U, and of S with V, and adding all of the results.

Gravity fields are frequently provided as normalized coefficients and must be converted for use in OrbGen. The fundamental relationship between the unnormalized and the normalized (bar) values, for the associated Legendre functions and the geopotential coefficients, is given by

$$\begin{aligned} &\bar{P}_{nm} = N_{nm} P_{nm} \qquad N_{nm} = \sqrt{\frac{(n-m)!(2n+1)(2-\delta_{0m})}{(n+m)!}} \\ &C_{nm} = N_{nm} \bar{C}_{nm} \qquad S_{nm} = N_{nm} \bar{S}_{nm} \\ &\delta_{0m} = 0 \quad m \neq 0 \qquad \delta_{00} = 1 \end{aligned} \tag{11.42}$$

$C_{00} = 1$ if the mass is correct and $C_{10} = C_{11} = S_{11} = 0$ if the origin is at the center of mass. $C_{21} = S_{21} = 0$ if the g' axis is along the principal moment of inertia. These values will be altered by the pole tide effect (see section 11.4.4, page 52). C_{20} could be non-zero and would contain the effect of the permanent tide if the tide model used in the determination of the gravity field did not contain that effect. EGM96, without a permanent tide effect in C_{20} (see section 11.4.3, page 52), and with an epoch of 1986, has values

$$\begin{aligned} C_{20} &= -0.10826266835532D-02 \quad \text{EGM96 unnormalized} \\ C_{21} &= -0.24140000000014D-09 \quad \text{EGM96 unnormalized} \\ S_{21} &= +0.15431000000045D-08 \quad \text{EGM96 unnormalized} \end{aligned} \tag{11.43}$$

$$\begin{aligned} \bar{C}_{20} &= -0.484165371736D-03 \quad \text{EGM96 normalized} \\ \bar{C}_{21} &= -0.186987635955D-09 \quad \text{EGM96 normalized} \\ \bar{S}_{21} &= +0.119528012031D-08 \quad \text{EGM96 normalized} \end{aligned} \tag{11.44}$$

Remark:

A_{earth} from 11.1, page 39 is assumed to be evaluated in the coordinate frame which is aligned with the mean inertia axis of the Earth. This axis is assumed to be determined as the six year

average position of the Celestial Ephemeris Pole which we will denote here as the $\overline{CEP}$ pole. It is assumed that the inertial position vector r has been transformed to $r_{\overline{CEP}}$ coordinates for evaluation in A_{earth}.

If the position of the $\overline{CEP}$ pole is given by coordinates (x_m, y_m) measured in the same way as the polar motion parameters $(\Delta x, \Delta y)$ of section 12.11.4, page 111, then the transformation from the $\overline{CEP}$ pole to the IRP pole frame will be similarly given by

$$A_p = \begin{bmatrix} 1 & 0 & x_m \\ 0 & 1 & -y_m \\ -x_m & y_m & 1 \end{bmatrix} \tag{11.45}$$

These six year average pole positions will include about five 430 day periods of the Chandler motion and are periodically published by the USNO. They can be obtained from the USNO web site *http://maia.usno.navy.mil/conv2000/chapter7/annual.pole.*

Putting the A_p transformation together with the polar motion transformation A

$$A = \begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & -\Delta y \\ -\Delta x & \Delta y & 1 \end{bmatrix} \tag{11.46}$$

gives the $A_p^T A$ transformation which takes coordinates in the CEP frame to coordinates in the $\overline{CEP}$ frame. The effect of neglecting this transformation when forming Earth-fixed coordinates (e', f', g') would be to alter the C_{21}, S_{21} gravity coefficients by setting

$$\begin{aligned} C_{21} &= C_{20}(x_m - \Delta x) \\ S_{21} &= -C_{20}(y_m - \Delta y) \end{aligned} \tag{11.47}$$

In [IERS96] it is assumed that only the A_p transformation has been neglected and that the A transformation was applied in the formation of (e', f', g') for the calculation of A_{earth}. Thus the expression for C_{21}, S_{21} does not show polar motion parameters as given above. The last equation would become

$$\begin{aligned} C_{21} &= C_{20} x_m & \bar{C}_{21} &= \sqrt(3)\bar{C}_{20} x_m \\ S_{21} &= -C_{20} y_m & \bar{S}_{21} &= -\sqrt(3)\bar{C}_{20} y_m \end{aligned} \tag{11.48}$$

The OrbGen program could be altered to include A in calculating (e', f', g') or it could alter C_{21}, S_{21} by

$$\begin{aligned} C_{21} &= C_{21}(\text{from Gravity Field, e.g., EGM96}) - C_{20}(\Delta x) \\ S_{21} &= S_{21}(\text{from gravity Field, e.g., EGM96}) - C_{20}(-\Delta y) \end{aligned} \tag{11.49}$$

If A is included in (e', f', g') then care must be taken to develop Earth orientation partials as discussed in section 12.11.5, page 112. If instead C_{21}, S_{21} are altered as in equation (11.49) then Earth orientation partials must be developed around that formula. This work will be necessary

in order to account for the effect of Earth orientation parameters on the orbit, in order that any subsequent estimation process for these parameters will have access to the orbital effect.

Finally it must be mentioned that the six year annual pole values from USNO, (x_m, y_m), are certainly time dependent values and would likely not be available when needed. This is handled, for example, in the EGM96 case above, with epoch of 1986, by providing rates for the C_{20}, C_{21}, S_{21} values on page C-1 of [EGM96].

These values and rates have been updated to 2000 and are given in Chapter 6 of the IERS Conventions 2000, *http://maia.usno.navy.mil/conv2000.html*.

The normalized values for EGM96, with epoch year 2000, taken from the reference above, are

$$\begin{aligned} \bar{C}_{20} &= -0.484165208950D-03 & \dot{\bar{C}}_{20} &= +1.162755D-11/year \\ \bar{C}_{21} &= -2.1954463704446D-10 & \dot{\bar{C}}_{21} &= -0.337D-11/year \\ \bar{S}_{21} &= +1.451433989350D-09 & \dot{\bar{S}}_{21} &= +1.606D-11/year \end{aligned} \tag{11.50}$$

where equation (11.48) would be used to calculate $\bar{C}_{21}$, $\bar{S}_{21}$. $\bar{C}_{20}$ would have been obtained by moving it from it's value at 1986, given in equation (11.44), using the rate $\dot{\bar{C}}_{20}$. The x_m, y_m mean pole values and their rates would have come from equation (11.78) in section 11.4.4, page 52, on Pole Tide. The rates $\dot{\bar{C}}_{21}$, $\dot{\bar{S}}_{21}$ are obtained by differentiating equation (11.48).

11.2 Sun-Moon Gravity

Accelerations due to the solar and lunar forces are given by

$$A_{sun} = -\mu_s \left(\frac{r - r_s}{|r - r_s|^3} + \frac{r_s}{|r_s|^3} \right) \tag{11.51}$$

$$A_{moon} = -\mu_m \left(\frac{r - r_m}{|r - r_m|^3} + \frac{r_m}{|r_m|^3} \right) \tag{11.52}$$

$$\tag{11.53}$$

Their partials with respect to the state are

$$\partial A_{sun}/\partial r = -\mu_s \left(\frac{I - 3\widehat{(r - r_s)}\widehat{(r - r_s)}^T}{|r - r_s|^3} \right) \tag{11.54}$$

$$\partial A_{sun}/\partial \dot{r} = 0 \tag{11.55}$$

$$\partial A_{moon}/\partial r = -\mu_m \left(\frac{I - 3\widehat{(r - r_m)}\widehat{(r - r_m)}^T}{|r - r_m|^3} \right) \tag{11.56}$$

$$\partial A_{moon}/\partial \dot{r} = 0 \tag{11.57}$$

11.3 Planetary Gravity

Force model calculations for Mercury, Venus, Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto are identical to those for the Moon. Coordinates for the planets are stored on the Sun-Moon file and accessed in the same way as the Moon's coordinates. Gravitational constants for the planets are provide on the general input file and the coordinates stored on the Sun-Moon file originally come from the JPL Planetary Ephemeris file. All or any subset of the planets can be used.

11.4 Tides

11.4.1 NSWCDD Solid-Earth Tide

This model, referred to as NSWCDD P2 [Groeger], expresses the disturbance to the Earth's gravitational field, due to the Sun and the Moon, with the potentials

$$\begin{aligned} U_s &= -K_l(\mu_s/|r_s|^3)(a_e^5/|r|^3)P_2(\hat{r}^T\hat{r}_s) \\ U_m &= -K_l(\mu_m/|r_m|^3)(a_e^5/|r|^3)P_2(\hat{r}^T\hat{r}_m) \\ P_2(x) &= (3x^2-1)/2 \quad \text{Legendre polynomial} \\ K_l &= \text{Love's constant} \end{aligned} \tag{11.58}$$

The associated disturbing acceleration due to the Sun and the Moon is given by

$$\begin{aligned} A_{tide} = {} & K_l(\mu_s/|r_s|^3)(a_e^5/|r|^5)\left[(-15(\hat{r}^T\hat{r}_s)^2+3)r/2+3r^T\hat{r}_s\hat{r}_s\right] \\ & +K_l(\mu_m/|r_m|^3)(a_e^5/|r|^5)\left[(-15(\hat{r}^T\hat{r}_m)^2+3)r/2+3r^T\hat{r}_m\hat{r}_m\right] \end{aligned} \tag{11.59}$$

The partials with respect to the state are

$$\begin{aligned} \partial A_{tide}/\partial r = {} & K_l/2(\mu_s/|r_s|^3)(a_e^5/|r|^5)\left[(-15(\hat{r}^T\hat{r}_s)^2+3)I\right. \\ & \left.+6\hat{r}_s\hat{r}_s{}^T+(105\hat{r}^T\hat{r}_s-15)\hat{r}\hat{r}^T-30\hat{r}^T\hat{r}_s(\hat{r}\hat{r}_s{}^T+\hat{r}_s\hat{r}^T)\right] \\ & K_l/2(\mu_m/|r_m|^3)(a_e^5/|r|^5)\left[(-15(\hat{r}^T\hat{r}_m)^2+3)I\right. \\ & \left.+6\hat{r}_m\hat{r}_m{}^T+(105\hat{r}^T\hat{r}_m-15)\hat{r}\hat{r}^T-30\hat{r}^T\hat{r}_m(\hat{r}\hat{r}_m{}^T+\hat{r}_m\hat{r}^T)\right] \end{aligned} \tag{11.60}$$

11.4.2 IERS Solid-Earth Tide

Tidal accelerations are categorized as solid-Earth tide, permanent tide, and pole tide. The permanent tide and pole tide effects are both solid-Earth tidal components but they will be described in their own sections. The resultant tidal accelerations, computed at each time line of the integration process, are summed together and added to the other accelerations. The models are documented in Chapter 6 of [IERS96], the Explanatory Supplement in [Schuh], and in an NSWCDD internal summary of these documents [Tanenbaum].

All of the tidal effects are calculated as time-dependent adjustments ΔC_{nm}, ΔS_{nm} to the unnormalized coefficients in a spherical harmonic expansion of the Earth's geopotential. These potential coefficient adjustments are used to evaluate the net tidal acceleration and position partial

derivatives at each time line. The formulas from section 11.1, page 39 can be used below, with the tide time-dependent adjustment coefficients, to express the tide potential and acceleration. These formulas can be used for all the tide acceleration computations once the $(\Delta C_{nm}, \Delta S_{nm})$ coefficient adjustments are evaluated. An alternative to calculating tidal accelerations and summing them into the total acceleration is to add the final $(\Delta C_{nm}, \Delta S_{nm})$ values into the gravity coefficients prior to calculating the gravity contribution to acceleration. This procedure is not used here. However, the harmonic functions (U_n^m, V_n^m) in the expression can be evaluated once and used to compute both the tidal and the geopotential accelerations.

$$V = \sum_{n=0}^{N} \sum_{m=0}^{n} \left[\Delta C_{nm} U_n^m + \Delta S_{nm} V_n^m \right] \tag{11.61}$$

$$A_{tide} = E^T a_e \nabla V = E^T \sum_{n=0}^{N} \sum_{m=0}^{n} \left[\Delta C_{nm} a_e \nabla U_n^m + \Delta S_{nm} a_e \nabla V_n^m \right] \tag{11.62}$$

The solid-Earth tide model currently has fewer than 100 terms with the contributions to the $(\Delta C_{nm}, \Delta S_{nm})$ coefficients developed in a two-step process followed by possible contributions for the permanent tide and the pole tide. Some of the contributions depend on choosing between an elastic (default) or an anelastic solid-Earth model. All contributions are to the degree 2, 3, and 4 harmonic coefficients. This work requires using the coefficients from the Coefficients for Fundamental Arguments J2000 and calculating the Fundamental Arguments in equation 11.63, page 48.

Coefficients for Fundamental Arguments J2000

l_0	485868.249036	D_0	1072260.703692
l_1	1717915923.2178	D_1	1602961601.2090
l_2	31.8792	D_2	-6.3706
l_3	.051635	D_3	.006593
l_4	-.00024470	D_4	-.00003169
l'_0	1287104.793048	Ω_0	450160.398036
l'_1	129596581.0481	Ω_1	-6962890.5431
l'_2	-.5532	Ω_2	7.4722
l'_3	.000136	Ω_3	.007702
l'_4	-.00001149	Ω_4	-.00005939
F_0	335779.5262320	G_0	361658.22615
F_1	1739527262.8478	G_1	129602772.19299
F_2	-12.7512	G_2	1.39656
F_3	-.001037	G_3	-9.3D-05
F_4	.00000417	G_4	.000

Fundamental Arguments (Delaunay Variables) (arcseconds)

$$
\begin{array}{ll}
l = \text{Mean anomaly of the Moon} & = l_0 + l_1 t + l_2 t^2 + l_3 t^3 + l_4 t^4 \\
l' = \text{Mean anomaly of the Sun} & = l'_0 + l'_1 t + l'_2 t^2 + l'_3 t^3 + l'_4 t^4 \\
F = L - \Omega & = F_0 + F_1 t + F_2 t^2 + F_3 t^3 + F_4 t^4 \\
D = \text{Elongation of the Moon from the Sun} & = D_0 + D_1 t + D_2 t^2 + D_3 t^3 + D_4 t^4 \\
\Omega = \text{Longitude of the Moon's mean node} & = \Omega_0 + \Omega_1 t + \Omega_2 t^2 + \Omega_3 t^3 + \Omega_4 t^4 \\
L = \text{Mean longitude of the Moon} & \text{Mod each value above with } S \\
G = GreenwichMeanSiderealTime & = G_0 + (36525.0S + G_1)t + G_2 t^2 \\
t = (MJD - MJD_{J2000})/36525.0 & + G_3 t^3 + G_4 t^4 \\
S = 1296000.000 &
\end{array}
\tag{11.63}
$$

Step 1 contributions for each harmonic, $(\Delta C_{nm}\ \Delta S_{nm})$, of degree 2 and 3, and some of degree 4 are given below. The formulas are the same, with the exception of body constants for the Moon, j=2, and the Sun, j=3.

Using IERS Technical Note 21 at [IERS96], using the fundamental relationship between normalized (bar) and unnormalized values, for the associated Legendre functions and the geopotential coefficients,

$$
\begin{array}{ll}
\bar{P}_{nm} = N_{nm} P_{nm} & N_{nm} = \sqrt{\dfrac{(n-m)!(2n+1)(2-\delta_{0m})}{(n+m)!}} \\
C_{nm} = N_{nm} \bar{C}_{nm} & S_{nm} = N_{nm} \bar{S}_{nm} \\
\delta_{0m} = 0 \quad m \neq 0 & \delta_{00} = 1
\end{array}
\tag{11.64}
$$

and using the Step 1 equations below, give, for degrees 2 and 3

$$
\begin{aligned}
\Delta C_{nm} &= \frac{N_{nm}^2}{(2n+1)} \frac{\mu_j}{\mu} \left(\frac{r_e}{r_j}\right)^{n+1} \left(\frac{1}{\mu r_e^n}\right) (Re(k_{nm}) U_n^m(\hat{r}_{je}) + Im(k_{nm}) V_n^m(\hat{r}_{je})) \\
\Delta S_{nm} &= \frac{N_{nm}^2}{(2n+1)} \frac{\mu_j}{\mu} \left(\frac{r_e}{r_j}\right)^{n+1} \left(\frac{1}{\mu r_e^n}\right) (Re(k_{nm}) V_n^m(\hat{r}_{je}) - Im(k_{nm}) U_n^m(\hat{r}_{je})) \\
U_n^m(\hat{r}_{je}) &= (\mu r_e^n) P_{nm}(cos(colatitude_j)) \cos(m \bullet longitude_j) \\
V_n^m(\hat{r}_{je}) &= (\mu r_e^n) P_{nm}(cos(colatitude_j)) \sin(m \bullet longitude_j)
\end{aligned}
\tag{11.65}
$$

U_n^m, V_n^m defined by equation 11.1, page 39.

The degree n=4 contributions are most easily computed from the degree 2 contributions by dividing out $N_{2m} k_{2m}$ and multiplying by $N_{4m} k_{2m}^{+}$ giving

$$
\begin{aligned}
\Delta C_{4m} &= N_{4m} k_{2m}^{+} \Delta C_{2m} / (N_{2m} k_{2m}) \\
\Delta S_{4m} &= N_{4m} k_{2m}^{+} \Delta S_{2m} / (N_{2m} k_{2m})
\end{aligned}
\tag{11.66}
$$

These calculations are made for the specific 10 pairs (n,m) of harmonic terms in the Step 1 Equations. The calculations are made for both the Sun and the Moon and summed together. Elastic and anelastic computations are made with the same formula with the only difference being that there is no imaginary component in the elastic case, i.e., $Re(k_{nm}) = k_{nm}$.

Letting

$$r_j = \text{Inertial position (j : 2,3 = Moon, Sun)}$$

$$r_{je} = \text{Earth-fixed position (j : 2,3 = Moon, Sun)}$$

$$r_e = \text{Earth radius}$$

$$\begin{bmatrix} x_{je} \\ y_{je} \\ z_{je} \end{bmatrix} = r_{je}/|r_{je}| \quad \text{Earth-fixed body(j) unit vector}$$

Step 1 Equations

(n,m)	$U_n^m(\hat{r}_{je})$	$V_n^m(\hat{r}_{je})$
$(2,0)$	$(\mu r_e^2)\left(\frac{3}{2}z_{je}^2 - \frac{1}{2}\right)$	0
$(2,1)$	$3(\mu r_e^2)\left(x_{je}z_{je}\right)$	$3(\mu r_e^2)\left(y_{je}z_{je}\right)$
$(2,2)$	$3(\mu r_e^2)\left(x_{je}^2 - y_{je}^2\right)$	$6(\mu r_e^2)\left(x_{je}y_{je}\right)$
$(3,0)$	$(\mu r_e^3)\left(\frac{5}{2}z_{je}^2 - \frac{3}{2}\right)z_{je}$	0
$(3,1)$	$(\mu r_e^3)\left(\frac{15}{2}z_{je}^2 - \frac{3}{2}\right)x_{je}$	$(\mu r_e^3)\left(\frac{15}{2}z_{je}^2 - \frac{3}{2}\right)y_{je}$
$(3,2)$	$15(\mu r_e^3)\left(x_{je}^2 - y_{je}^2\right)z_{je}$	$30(\mu r_e^3)\left(x_{je}y_{je}z_{je}\right)$
$(3,3)$	$15(\mu r_e^3)\left(x_{je}^3 - 3x_{je}y_{je}^2\right)$	$15(\mu r_e^3)\left(3x_{je}^2y_{je} - y_{je}^3\right)$

k_{nm} Values

	Elastic Earth		**Anelastic Earth**		
(n,m)	k_{nm}	k_{2m}^{+}	$Re(k_{nm})$	$Im(k_{nm})$	k_{2m}^{+}
$(2,0)$	$0.29525D0$	$-.00087D0$	$0.30190D0$		$-.00089D0$
$(2,1)$	$0.29470D0$	$-.00079D0$	$0.29830D0$	$-.00144D0$	$-.00080D0$
$(2,2)$	$0.29801D0$	$-.00057D0$	$0.30102D0$	$-.00130D0$	$-.00057D0$
$(3,0)$	$0.09300D0$		$0.09300D0$		
$(3,1)$	$0.09300D0$		$0.09300D0$		
$(3,2)$	$0.09300D0$		$0.09300D0$		
$(3,3)$	$0.09400D0$		$0.09400D0$		

To implement the NSWCDD P2 tidal model in this formulation, use only the harmonic coefficient contributions for terms of degree 2 and substitute the Love number, k_{Love}, for the tabulated values of k_{2m}. The result of this work is the $(\Delta C_{nm}(Step1), \Delta S_{nm}(Step1))$ contribution to the $(\Delta C_{nm}, \Delta S_{nm})$ coefficients in 11.61, page 47.

Step 2 Elastic/Anelastic contributions to the harmonic coefficients $(\Delta C_{nm}\ \Delta S_{nm})$. After evaluating the polynomials and finding the fundamental arguments $l,\ l',\ F,\ D,\ \Omega,$ and G above, also called Delaunay variables, compute a second set of fundamental angles $\tau,\ s,\ h,\ p,\ N',$ and ps or β_1 through β_6, also called Doodson's variables. These angles will be used with the IERS Solid-Earth Tabular Data on page 56. The Doodson's variables are related to the Delaunay variables by

Fundamental Angles (Doodson's Variables)

$$\begin{array}{ll} \beta_1 = -F - \Omega + G + 648000'' & \beta_4 = +F + \Omega - L \\ \beta_2 = +F + \Omega & \beta_5 = -\Omega \\ \beta_3 = +F + \Omega - D & \beta_6 = +F + \Omega - D - L' \\ S = 1296000.000 & \beta_i = mod(2\pi/S\beta_i, 2\pi) \end{array} \tag{11.67}$$

$\beta_1 =$ Greenwich mean lunar time
$\beta_2 =$ Mean longitude of the moon
$\beta_3 =$ Mean longitude of the sun
$\beta_4 =$ Mean longitude of the lunar perigee
$\beta_5 =$ Negative mean longitude of the lunar node
$\beta_6 =$ Mean longitude of the solar perigee

The importance of these calculations is that each tidal disturbing term (s) is associated with its Doodson's number D_s, its time-varying angular argument θ_s, and some amplitudes. The θ_s and D_s are computed by

$$\theta_s = \sum_{i=1}^{6} \beta_i d_{is} \tag{11.68}$$

$$D_s = \sum_{i=1}^{6} (1/10)^{(i-3)}(d_{is} + 5) - 500 \tag{11.69}$$

where the Doodson's multipliers, d_{is}, obtained from the IERS Solid-Earth Tabular Data on page 56, are opposite their Doodson's number, for each disturbing term (s). The Doodson's numbers are in column 1, the Doodson's multipliers, d_{is}, are the first six values appearing in columns (3-13), below the Doodson's variable names, and the amplitudes are in columns 15 and 17. For each Doodson's number D_s the contribution to ΔC_{nm} and ΔS_{nm} depends on the choice of an anelastic or elastic solid-Earth tide option.

The tabular data is separated into 3 blocks : order 0 (C_{20}) coefficients (long-period tides), order 1 (C_{21} S_{21}) coefficients (diurnal tides), and order 2 (C_{22} S_{22}) coefficients (semi-diurnal tides). The first block contains 21 terms, each of which has specified : a Doodson's number and the in-phase amplitudes Aip_s, in the column labelled Aip, and out-of-phase amplitudes Aop_s in the column labelled Aop. These values are used to compute the anelastic contribution to ΔC_{20}, with the elastic contribution being zero. The second block contains 26 terms, each with Doodson's number and two sets of amplitudes A_s^{el} and A_s^{anel}, one under column Ael is for the elastic case, and the other Aanel is for the anelastic case. These values are used to compute elastic or anelastic contributions to (C_{21}, S_{21}). The third block contains two terms, each specifying a Doodson's number and an amplitude Amp_s, under column Amp, for the contribution to (C_{22}, S_{22}). The tabular values need to be scaled, so setting the scale factor $F = 10^{-12}$ gives

$$\begin{array}{ll} \text{Anelastic Case} & \text{Elastic Case} \\ \Delta C_{20} = FN_{20}\sum_s (Aip_s \cos(\theta_s) + Aop_s \sin(\theta_s)) & \Delta C_{20} = 0 \\ N_{20} = \sqrt{5} & \end{array} \tag{11.70}$$

$$\begin{array}{ll} \text{Anelastic Case} & \text{Elastic Case} \\ \Delta C_{21} = FN_{21}\sum_s (A_s^{anel} \sin(\theta_s)) & \Delta C_{21} = FN_{21}\sum_s (A_s^{el} \sin(\theta_s)) \\ \Delta S_{21} = FN_{21}\sum_s (A_s^{anel} \cos(\theta_s)) & \Delta S_{21} = FN_{21}\sum_s (A_s^{el} \cos(\theta_s)) \\ N_{21} = \sqrt{5/3} & N_{21} = \sqrt{5/3} \end{array} \tag{11.71}$$

where the angle θ_s is defined as above, using the Doodson's variables and multipliers at the time of computation.

One additional contribution, for Doodson's number 165555, anelastic case, is specifically given as

$$\begin{array}{ll} \text{Anelastic Case} & \text{Elastic Case} \\ \Delta C_{21} = +N_{21}(0.46\text{D-}12)\cos(\theta_s) & \Delta C_{21} = 0 \\ \Delta S_{21} = -N_{21}(0.46\text{D-}12)\sin(\theta_s) & \Delta S_{21} = 0 \end{array} \tag{11.72}$$

Contributions to C_{22} and S_{22} are computed for only 2 tabulated constituents (s) and are identical for the elastic and the anelastic cases. This means that only one set of amplitudes Amp_s needs to be provided for each constituent.

$$\begin{array}{ll} \text{Anelastic Case} & \text{Elastic Case} \\ \Delta C_{22} = FN_{22}\sum_s Amp_s \cos(\theta_s) & \Delta C_{22} = FN_{22}\sum_s Amp_s \cos(\theta_s) \\ \Delta S_{22} = FN_{22}\sum_s Amp_s \sin(\theta_s) & \Delta S_{22} = FN_{22}\sum_s Amp_s \sin(\theta_s) \\ N_{22} = \sqrt{5/12} & N_{22} = \sqrt{5/12} \end{array} \tag{11.73}$$

The result of this work is the $(\Delta C_{nm}(Step2), \Delta S_{nm}(Step2))$ contribution to the $(\Delta C_{nm}, \Delta S_{nm})$ coefficients in 11.61, page 47. Adding the Step 1 and 2 contributions, together with the permanent tide and pole tide contributions, if required, gives the total $(\Delta C_{nm}, \Delta S_{nm})$ values.

11.4.3 Permanent Tide

The permanent tide (Honkisalo) correction can be used with both the IERS 96 and the NSWCDD P2 model. The permanent tide correction, here, is independent of tidal model choice, and depends only on the gravity model choice. This is because the tide model choices, here, include only models which have included the permanent tide effect on the satellite. If the gravity model used has the permanent tide included, which means that the gravity model was determined in conjunction with a tide model that did not include the permanent tide effect, then the permanent tide effect in our tide model, here, must be removed, in order to avoid including the effect twice.

The way to remove the permanent tide effect from our tide model, here, is to include the permanent tide correction. The meaning of the permanent tide correction is to remove the permanent tide contribution from our tide model. This will ensure that the contribution of the permanent tide will only be included once.

A gravity model should state that it includes the permanent tide effect or not, but it would be indicated by looking at the C_{20} gravity coefficient and comparing it to one from a gravity field which did not include the permanent tide effect, e.g., [EGM96] with epoch 1986.

$$\begin{aligned} C_{20} &= -0.10826266835532D-02 \quad \text{EGM96 unnormalized} \\ \bar{C}_{20} &= -0.484165371736D-03 \quad\quad \text{EGM96 normalized} \end{aligned}$$

Setting $k_{20} = k_{Love}$ by model

	Anelastic	Elastic	NSWCDD P2
k_{Love}	$0.30190D0$	$0.29525D0$	$0.29000D0$

gives a permanent tide correction of

$$\Delta C_{20} = N_{20}(4.4228\text{D-}08)(0.31460\text{D+}00)k_{\text{Love}} \tag{11.74}$$

11.4.4 Pole Tide

The pole tide correction should only be used with the IERS tide model. The pole tide correction depends upon instantaneous values $(\Delta x, \Delta y)$ and a mean polar displacement (x_m, y_m). The instantaneous polar motion values $(\Delta x, \Delta y)$ and (x_m, y_m) should be evaluated at each time of computation.

The mean pole can be defined, approximately, as the extrapolated secular mean pole, using the linear model

$$x_m = x_{m0} + x_{mr}(Y - Y_o) \quad Y_o = \text{reference year} \tag{11.75}$$

$$y_m = y_{m0} + y_{mr}(Y - Y_o) \quad Y = \text{time of computation} \tag{11.76}$$

The predicted mean pole model consists of 5 values $(x_{mo}, y_{mo}, x_{mr}, y_{mr}, Y_o)$ which are derived from published data. The values below, with epoch 1950, were recommended by USNO, and are in the current operational program. The IERS has updated values below, with epoch 2000, that

appear in Chapter 7 of the IERS Conventions 2000, *http://maia.usno.navy.mil/conv2000.html*, with the published values located at *http://maia.usno.navy.mil/conv2000/chapter7/annual.pole*. The straight line must be used as the six year average tabular values will always lag current time.

$$x_{mo} = -0.7 \quad y_{mo} = 179.4 \quad x_{mr} = 0.862 \quad y_{mr} = 3.217 \quad Y_o = 1950 \tag{11.77}$$

$$x_{mo} = 54.0 \quad y_{mo} = 357.0 \quad x_{mr} = 0.830 \quad y_{mr} = 3.950 \quad Y_o = 2000 \tag{11.78}$$

Current program coefficient values are obtained from the Predicted Mean Pole Values file on page 152.

Pole values are given in milliarcseconds (mas) and mas/yr, and multiplying by the conversion factor ($\pi/0.648\text{D+}09$) converts from mas to radians. The pole tide correction given here is in terms of polar motion values (Δx, Δy, x_m, y_m) in radians. This correction for the unnormalized harmonics follows.

Pole Tide Correction

$$\begin{aligned}
&\text{Elastic Case}\\
\Delta C_{21} &= -3.435099\text{D-}04(\Delta x - x_m)\\
\Delta S_{21} &= +3.435099\text{D-}04(\Delta y - y_m)\\
&\text{Anelastic Case}\\
\Delta C_{21} &= -3.589545\text{D-}04((\Delta x - x_m) + 0.0112(\Delta y - y_m))\\
\Delta S_{21} &= +3.589545\text{D-}04((\Delta y - y_m) - 0.0112(\Delta x - x_m))
\end{aligned} \tag{11.79}$$

The pole tide component of equation 11.62, page 47, can now be expressed as

$$A_{poletide} = E^T \left[\Delta C_{21} a_e \nabla U_2^1 + \Delta S_{21} a_e \nabla V_2^1 \right] \tag{11.80}$$

using only the ΔC_{21}, ΔS_{21} contributing terms above.

When comparing with the IERS Conventions remember that their formulas were given in units of arcseconds for correction of normalized geopotential harmonics $\overline{C}_{21}$ and $\overline{S}_{21}$.

11.4.5 IERS Ocean Tide

The dynamical effects of ocean tides are most easily incorporated by periodic variations in the unnormalized Stokes' coefficients. Using IERS Technical Note 21 at [IERS96], these variations can be written as

$$\begin{aligned}
\Delta C_{nm} &= F_{nm} \sum_{s(n,m)} [(C^+_{snm} + C^-_{snm})\cos\theta_s + (S^+_{snm} + S^-_{snm})\sin\theta_s]\\
\Delta S_{nm} &= F_{nm} \sum_{s(n,m)} [(S^+_{snm} - S^-_{snm})\cos\theta_s - (C^+_{snm} - C^-_{snm})\sin\theta_s]\\
F_{nm} &= \frac{4\pi G\rho_w}{g}\left(\frac{1+k'_n}{2n+1}\right)
\end{aligned} \tag{11.81}$$

$g = 9.780327D+02\ cm\ s^{-2}$ is the Earth mean equatorial gravity.

$G = 6.67259D-05\ cm^3\ kg^{-1}\ s^{-2}$ is the universal constant of gravitation.

$\rho_w = 1.025D-03\ kg\ cm^{-3}$ is the density of seawater.

k'_n are load deformation coefficients from the ocean tide file.

$\theta_s = \sum_{i=1}^{6} \beta_i d_{is}$ is the angular argument of the tide constituent s, as defined in equation 11.68, page 50, in the IERS Solid-Earth Tide section.

$C^{\pm}_{snm}, S^{\pm}_{snm}$ are ocean tide coefficients (cm) for the tide constituent s, from the ocean tide file. The line for constituent s has the coefficients in the order C^{+}_{snm}, S^{+}_{snm}, C^{-}_{snm}, S^{-}_{snm}. The summation over + and - denotes the respective addition of the retrograde waves using the + sign and the prograde waves using the - sign. The $C^{\pm}_{snm}$ and $S^{\pm}_{snm}$ are the coefficients of a spherical harmonic decomposition of the ocean tide height, for the ocean tide, due to the constituent s of the tide generating potential. The ocean tide file can be obtained via ftp with the following address. *ftp://ftp.csr.utexas.edu/pub/grav/OTIDES.TOPEX_3.0.*

For each constituent s in the diurnal and semi-diurnal tidal bands, these coefficients were obtained from the CSR 3.0 ocean tide height model, [Eanes]. For each constituent s in the long period band, the self-consistent equilibrium tide model [Cartwright] was used.

The resonant harmonics for some constituents were determined by their combined effects on the orbits of several satellites. These multi-satellite values then replaced the corresponding values from the CSR 3.0 ocean tide height model, the most recent solution for ocean tides obtained with the TOPEX/POSEIDON altimeter data. This replacement is done by using the TEG-2B correction terms below.

The ocean tide data file, obtained from the University of Texas, begins with a header section consisting of one line of text. The next line contains the integer numbers $nx1$, $nx2$, nxt, mxt (4I4), which are used to read in the one small and three large data blocks that follow.

The small data block has $nxt+5$ floating point constants (5E21.14/(6e21.14)) and at present, the first six fields are not being used. The values nxt and mxt are the maximum degree and order of the tidal expansion, which was calculated at the University of Texas in preparing the file. The next $nxt-1$ values are the load deformation coefficients, k'_n $n = 2, 3, \ldots nxt$.

The first of the large data blocks contains $nx1$ one-line records . Each record corresponds to one tidal constituent s, which is identified by its Doodson's number D_s. These entries are followed by an identifier, an integer, and four floating point values. The first floating point value , Hs, is used in a test to determine which of the constituents in the second large data block are actually to be used in computing contributions to equation 11.81, page 53. Before testing constituents, the second large data block is altered by using the TEG-2B correction terms, in the last data block, described below. Frequency, phase offset, and Wahr Ks are the remaining three unused values (F8.3, A4, 1x, I3, 4x, 4E21.14).

The Doodson numbers of the tidal constituents in the first large data block having $|Hs| > H_{min}$ are labeled "good", by the test. A constituent will subsequently be used if it's Doodson's number is "good".

The second large data block contains $nx2$ one-line records . Each record contains, in addition to a Doodson's number, D_s, a four-character identifier, ID_s, an n, m pair, for degree and order,

and associated coefficients C^{+}_{snm}, S^{+}_{snm}, C^{-}_{snm}, S^{-}_{snm} (a10,i2, f8.3,a4, 2i2,2x,4e22.14,i2). Currently there are nearly 2000 records in this block, of which at least 600 would be "good", and testing may indicate that all terms are "good".

Each record in the last block, consists of TEG-2B correction terms. This last data block has the same format as the second large data block. Each constituent in this data block, with a given Doodson's number and n, m pair, is to replace a matching constituent in the second large data block. Matching will mean that the constituents have the same Doodson's number and the same n, m pair.

There could be several constituents in this last data block with the same Doodson's number and n, m pair. They will each be used, in the order that they appear, to replace the corresponding constituent in the second large data block. This would be equivalent to using only the last constituent, with a given Doodson's number and n, m pair, in this last data block, to do the replacement. It would also be equivalent to sorting the entire second and last data block by Doodson's number, n, m pair, and line number, and retaining only the last constituent of a given Doodson's number, and n, m pair.

Finally, only those constituents with "good" Doodson's numbers would be retained for contribution to equation 11.81, page 53.

11.4.6 Station Pole Tide

In performing the associated station pole tide correction (not used in OrbGen), the following formulas apply, at each time of computation.

$$\begin{bmatrix} E \\ N \\ U \end{bmatrix} = factor \begin{bmatrix} 9\sin(\phi)\sin(\lambda) & 9\sin(\phi)\cos(\lambda) \\ -9\cos(2\phi)\cos(\lambda) & 9\cos(2\phi)\sin(\lambda) \\ -32\sin(2\phi)\cos(\lambda) & 32\sin(2\phi)\sin(\lambda) \end{bmatrix} \begin{bmatrix} (\Delta x - x_m) \\ (\Delta y - y_m) \end{bmatrix} \tag{11.82}$$

where (E, N, U) are the station corrections, in kilometers, in the East, North, and Up directions. $factor = 0.648/\pi$, and $\lambda, and\ \phi$ are the station longitude and latitude, in radians, at the computation time. The daily and mean polar motion values $(\Delta x, \Delta y)$ and (x_m, y_m) are in radians, and are also at the computation time. They are described in section 11.4.4, page 52.

The coordinate corrections, (E, N, U) can be converted to cartesian corrections

$$\begin{bmatrix} dx_s \\ dy_s \\ dz_s \end{bmatrix} = \begin{bmatrix} -\sin(\lambda) & -\sin(\phi)\cos(\lambda) & \cos(\phi)\cos(\lambda) \\ \cos(\lambda) & -\sin(\phi)\sin(\lambda) & \cos(\phi)\sin(\lambda) \\ 0 & \cos(\phi) & \sin(\phi) \end{bmatrix} \begin{bmatrix} E \\ N \\ U \end{bmatrix} \tag{11.83}$$

and added to the cartesian Earth-fixed coordinates, (x_s, y_s, z_s), of the tracking station, before rotation to the Inertial frame. This task would be encountered when placing the satellite and the tracking station in a common frame of reference. Because the corrections are so small, it is also possible to accurately convert (E, N, U) perturbations into changes $(d\lambda, d\phi, dh)$, and to use the corrected (λ, ϕ, h) in the conversion from geodetic to cartesian coordinates of the station.

IERS Solid-Earth Tabular Data

2 0 0.30190 0.29525 (nominal; elastic; anelastic (real, imag) k20

dood#	deg/hr	τ	s	h	p	N'	ps	l	l'	F	D	Ω	δk_f^R	Aip	δk_f^I	Aop
55.565	0.00221	0	0	0	0	1	0	0	0	0	0	1	0.01347	16.6	-0.00541	-6.7
55.575	0.00441	0	0	0	0	2	0	0	0	0	0	2	0.01124	-0.1	-0.00488	0.1
56.554	0.04107	0	0	1	0	0	-1	0	-1	0	0	0	0.00547	-1.2	-0.00349	0.8
57.555	0.08214	0	0	2	0	0	0	0	0	-2	2	-2	0.00403	-5.5	-0.00315	4.3
57.565	0.08434	0	0	2	0	1	0	0	0	-2	2	-1	0.00398	0.1	-0.00313	-0.1
58.554	0.12320	0	0	3	0	0	-1	0	-1	-2	2	-2	0.00326	-0.3	-0.00296	0.2
63.655	0.47152	0	1	-2	1	0	0	1	0	0	-2	0	0.00101	-0.3	-0.00242	0.7
65.445	0.54217	0	1	0	-1	-1	0	-1	0	0	0	-1	0.00080	0.1	-0.00237	-0.2
65.455	0.54438	0	1	0	-1	0	0	-1	0	0	0	0	0.00080	-1.2	-0.00237	3.7
65.465	0.54658	0	1	0	-1	1	0	-1	0	0	0	1	0.00079	0.1	-0.00237	-0.2
65.655	0.55366	0	1	0	1	0	0	1	0	-2	0	-2	0.00077	0.1	-0.00236	-0.2
73.555	1.01590	0	2	-2	0	0	0	0	0	0	-2	0	-0.00009	0.0	-0.00216	0.6
75.355	1.08875	0	2	0	-2	0	0	-2	0	0	0	0	-0.00018	0.0	-0.00213	0.3
75.555	1.09804	0	2	0	0	0	0	0	0	-2	0	-2	-0.00019	0.6	-0.00213	6.3
75.565	1.10024	0	2	0	0	1	0	0	0	-2	0	-1	-0.00019	0.2	-0.00213	2.6
75.575	1.10245	0	2	0	0	2	0	0	0	-2	0	0	-0.00019	0.0	-0.00213	0.2
83.655	1.56956	0	3	-2	1	0	0	1	0	-2	-2	-2	-0.00065	0.1	-0.00202	0.2
85.455	1.64241	0	3	0	-1	0	0	-1	0	-2	0	-2	-0.00071	0.4	-0.00201	1.1
85.465	1.64462	0	3	0	-1	1	0	-1	0	-2	0	-1	-0.00071	0.2	-0.00201	0.5
93.555	2.11394	0	4	-2	0	0	0	0	0	-2	-2	-2	-0.00102	0.1	-0.00193	0.2
95.355	2.18679	0	4	0	-2	0	0	-2	0	-2	0	-2	-0.00106	0.1	-0.00192	0.1
		-1														

2 1 0.29470 0.29830 0.00144 (nominal; elastic; anelastic (real, imag) k21

dood#	deg/hr	τ	s	h	p	N'	ps	l	l'	F	D	Ω	δk_f^{el}	Ael	δk_f^{anel}	Aanel
135.645	13.39645	1	-2	0	1	-1	0	1	0	2	0	1	-0.00044	-0.1	-0.00045	-0.1
135.655	13.39866	1	-2	0	1	0	0	1	0	2	0	2	-0.00044	-0.7	-0.00046	-0.7
137.455	13.47151	1	-2	2	-1	0	0	-1	0	2	2	2	-0.00047	-0.1	-0.00049	-0.1
145.545	13.94083	1	-1	0	0	-1	0	0	0	2	0	1	-0.00081	-1.2	-0.00082	-1.3
145.555	13.94303	1	-1	0	0	0	0	0	0	2	0	2	-0.00081	-6.6	-0.00082	-6.7
153.655	14.41456	1	0	-2	1	0	0	1	0	2	-2	2	-0.00167	0.1	-0.00168	0.1
155.455	14.48741	1	0	0	-1	0	0	-1	0	2	0	2	-0.00193	0.4	-0.00193	0.4
155.655	14.49669	1	0	0	1	0	0	1	0	0	0	0	-0.00196	1.3	-0.00197	1.3
155.665	14.49890	1	0	0	1	1	0	1	0	0	0	1	-0.00197	0.2	-0.00198	0.3
157.455	14.56955	1	0	2	-1	0	0	-1	0	0	2	0	-0.00231	0.3	-0.00231	0.3
162.556	14.91787	1	1	-3	0	0	1	0	1	2	-2	2	-0.00834	-1.9	-0.00832	-1.9
163.545	14.95673	1	1	-2	0	-1	0	0	0	2	-2	1	-0.01114	0.5	-0.01111	0.5
163.555	14.95893	1	1	-2	0	0	0	0	0	2	-2	2	-0.01135	-43.3	-0.01132	-43.2
164.556	15.00000	1	1	-1	0	0	1	0	1	0	0	0	-0.01650	2.0	-0.01642	2.0
165.545	15.03886	1	1	0	0	-1	0	0	0	0	0	-1	-0.03854	-8.8	-0.03846	-8.8
165.555	15.04107	1	1	0	0	0	0	0	0	0	0	0	-0.04093	472.0	-0.04085	471.0
165.565	15.04328	1	1	0	0	1	0	0	0	0	0	1	-0.04365	68.3	-0.04357	68.2
165.575	15.04548	1	1	0	0	2	0	0	0	0	0	2	-0.04678	-1.6	-0.04670	-1.6
166.554	15.08214	1	1	1	0	0	-1	0	-1	0	0	0	0.23083	-20.8	0.22609	-20.4
167.555	15.12321	1	1	2	0	0	0	0	0	-2	2	-2	0.03051	-5.0	0.03027	-5.0
173.655	15.51259	1	2	-2	1	0	0	1	0	0	-2	0	0.00374	-0.5	0.00371	-0.5
175.455	15.58545	1	2	0	-1	0	0	-1	0	0	0	0	0.00329	-2.1	0.00325	-2.1
175.465	15.58765	1	2	0	-1	1	0	-1	0	0	0	1	0.00327	-0.4	0.00324	-0.4
183.555	16.05697	1	3	-2	0	0	0	0	0	0	-2	0	0.00198	-0.2	0.00195	-0.2
185.555	16.13911	1	3	0	0	0	0	0	0	-2	0	-2	0.00187	-0.7	0.00184	-0.6
185.565	16.14131	1	3	0	0	1	0	0	0	-2	0	-1	0.00187	-0.4	0.00184	-0.4
		-1														

2 2 0.29801 0.30102 0.00130 (nominal; elastic; anelastic (real, imag) k22

dood#	deg/hr	τ	s	h	p	N'	ps	l	l'	F	D	Ω	δk_f^R	Amp
245.655	28.43973	2	-1	0	1	0	0	1	0	2	0	2	0.00006	-0.3
255.555	28.98410	2	0	0	0	0	0	0	0	2	0	2	0.00004	-1.2
		-1												

ftp://maia.usno.navy.mil/conventions/chapter6/isc6.tex

11.5 Thrust

Acceleration due to thrust is defined by a thrust profile which provides thrust start and stop times. The thrust itself at time t, $T^b(t)$, is constant in some local body frame and rotated to the inertial frame by the transformation L.

If the components of thrust are represented by $(T_1^b,\ T_2^b,\ T_3^b)$ in the local thrust frame and the local thrust frame unit vectors are represented in the inertial frame by the column vectors L_1, L_2, L_3 then the inertial components of the thrust, A_{thrust}, will be given by

$$A_{thrust} = LT^b(t) = \begin{bmatrix} L_1 & L_2 & L_3 \end{bmatrix} \begin{bmatrix} T_1^b \\ T_2^b \\ T_3^b \end{bmatrix} \tag{11.84}$$

11.5.1 Frames and Transformations

The transformation L can be either of the three given below, where each transformation represents a rotation from the given local frame to the inertial frame.

(RVC) The historical CELEST Radial, Velocity, Cross-track frame.

$$L_1 = \hat{r} = r/|r| \quad L_2 = \hat{\dot{r}} = \dot{r}/|\dot{r}| \quad L_3 = \widehat{r \times \dot{r}} = (r \times \dot{r})/|r \times \dot{r}| \tag{11.85}$$

(RAC) The standard Radial, Along-track, Cross-track frame.

$$L_1 = \hat{r} \quad L_2 = \widehat{r \times \dot{r}} \times \hat{r} \quad L_3 = \widehat{r \times \dot{r}} \tag{11.86}$$

(GPS) The GPS Body (XYZ) frame.

$$\begin{aligned} L_1 = x_b = y_b \times z_b \quad L_2 = y_b = \widehat{z_b \times r_s} \quad L_3 = z_b = -\hat{r} \\ \text{If} \quad \sin(z_b, r_s) = |z_b \times r_s|/(|z_b||r_s|) < \varepsilon \\ \text{Then} \quad L_2 = \widehat{\dot{z}_b \times r_s} = \widehat{-\dot{r} \times r_s} \end{aligned} \tag{11.87}$$

The value ε is a tolerance to measure when the z_b axis is aligned with the Earth-Sun vector, r_s, and $\hat{v}$ is the unit vector associated with the vector v.

11.5.2 Frame Partials

$$\Omega_v = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} \quad \text{for any vector } v.$$

(RVC) The historical CELEST Radial, Velocity, Cross-track frame.

$$\begin{aligned}
\partial L_1/\partial r &= (I - L_1 L_1^T)/|r| \\
\partial L_1/\partial \dot{r} &= 0 \\
\partial L_2/\partial r &= 0 \\
\partial L_2/\partial \dot{r} &= (I - L_2 L_2^T)/|\dot{r}| \\
\partial L_3/\partial r &= (L_3 L_3^T - I)\Omega_{\dot{r}}/|\Omega_{\dot{r}} r| \\
\partial L_3/\partial \dot{r} &= (I - L_3 L_3^T)\Omega_r/|\Omega_r \dot{r}|
\end{aligned} \tag{11.88}$$

(RAC) The standard Radial, Along-track, Cross-track frame.

$$\begin{aligned}
\partial L_1/\partial r &= (I - L_1 L_1^T)/|r| \\
\partial L_1/\partial \dot{r} &= 0 \\
\partial L_2/\partial r &= \Omega_{L_3} \partial L_1/\partial r - \Omega_{L_1} \partial L_3/\partial r \\
\partial L_2/\partial \dot{r} &= -\Omega_{L_1} \partial L_3/\partial \dot{r} \\
\partial L_3/\partial r &= (L_3 L_3^T - I)\Omega_{\dot{r}}/|\Omega_{\dot{r}} r| \\
\partial L_3/\partial \dot{r} &= (I - L_3 L_3^T)\Omega_r/|\Omega_r \dot{r}|
\end{aligned} \tag{11.89}$$

(GPS) The GPS Body (XYZ) frame.

$$\begin{aligned}
\partial L_1/\partial r &= \Omega_{L_2} \partial L_3/\partial r - \Omega_{L_3} \partial L_2/\partial r \\
\partial L_1/\partial \dot{r} &= 0 \\
\partial L_2/\partial r &= (I - (\widehat{\Omega_{r_s} L_3})(\widehat{\Omega_{r_s} L_3})^T)(\Omega_{r_s}/(|r||\Omega_{r_s} L_3|)) \\
\partial L_2/\partial \dot{r} &= 0 \\
\partial L_3/\partial r &= (L_3 L_3^T - I)/|r| \\
\partial L_3/\partial \dot{r} &= 0
\end{aligned} \tag{11.90}$$

11.5.3 Parameter Partials

The partial derivatives with respect to position, velocity, and the thrust acceleration parameters, (T_1, T_2, T_3), are given by

$$\partial A_{thrust}/\partial r = (\partial L/\partial r)T^b \tag{11.91}$$

$$\partial A_{thrust}/\partial \dot{r} = (\partial L/\partial \dot{r})T^b \tag{11.92}$$

$$\partial A_{thrust}/\partial T^b = L \tag{11.93}$$

11.6 Drag

11.6.1 Acceleration

The relative velocity of the satellite with respect to the atmosphere is given by

$$v_r = \dot{r} - \vec{\omega} \times r = \dot{r} - \Omega_{\vec{\omega}} r \tag{11.94}$$

$$\text{where} \quad \vec{\omega} = (CD)^T \begin{bmatrix} 0 \\ 0 \\ \tilde{\omega} \end{bmatrix} = (CD)^T \omega$$

$$\tilde{\omega} = \text{Earth's mean sidereal rate (radians/second)} \tag{11.95}$$
$$= .72921158553D-04$$

$$\text{and} \quad \Omega_v = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} \quad \text{for any vector } v.$$

$$\text{Since} \quad \Omega_{Tv} = T\Omega_v T^T \quad \text{for } T \text{ orthogonal and any vector } v.$$

$$\Omega_{\vec{\omega}} = \Omega_{(CD)^T\omega} = (CD)^T \Omega_\omega (CD)$$

$$v_r = \dot{r} - (CD)^T \Omega_\omega (CD) r \tag{11.96}$$

The acceleration due to atmospheric drag can now be expressed as

$$A_{drag} = -(1/2) C_d \rho (s/m) |v_r| v_r \tag{11.97}$$

where m is the satellite mass in kilograms, s is the satellite cross-sectional area in square kilometers, and ρ is the atmospheric density function in kilograms per cubic kilometer. The atmospheric density function can be provided by any one of three models.

- **Jacchia-Bass 1977(modified)** [Jacchia], [Bass] The Jacchia 1977 model, modified by Bass (1980), incorporates density variation due to solar ultraviolet radiation, characterized by the solar flux at 10.7 cm wavelength, $F_{10.7}$, and an average value $\bar{F}_{10.7}$. These values indicate sunspot activity. Values in excess of 300 are indicative of extensive sunspot activity and values below 100 indicate very little activity. The model depends on variation in the Earth's magnetic field characterized by the planetary geomagnetic index K_p, developed by Julius Bartels [Bartels]. There is dependence on the time of year, time of day, height, h, above the Earth, and the Earth-fixed position of the satellite and the Sun. A file consisting of daily entries of a daily $F_{10.7}$ value, a 90-day average $\bar{F}_{10.7}$ value, and eight 3-hour K_p values are required to employ the model. Linear interpolation is used. There is an option to use a_p values in place of the K_p values. The file structure is the same but the values are different. This is indicated by an input value "kpuse". If this value is 0 then the values on the file are a_p values and they must be converted to K_p values before interpolation at the requested time. The conversion is as follows.

  In order to convert the value a_p to the corresponding K_p value define a function AKP such that the function value $K_p = AKP(a_p)$ is constructed as follows.

Define a linear array AP of 27 monotonically increasing values

$$AP = \begin{array}{ccccccccc} 2 & 3 & 4 & 5 & 6 & 7 & 9 & 12 & 15 \\ 18 & 22 & 27 & 32 & 39 & 48 & 56 & 67 & 80 \\ 94 & 111 & 132 & 154 & 179 & 207 & 236 & 300 & 400 \end{array} \qquad (11.98)$$

locate the first index i in the AP array such that $a_p < AP(i)$.

$$\begin{array}{ll} a_p < AP(1) & K_p = a_p/(AP(1)) \\ a_p \geq AP(27) & K_p = 27 \end{array}$$

Otherwise

$$K_p = (i-1) + (a_p - AP(i-1))/(AP(i) - AP(i-1)) \qquad (11.99)$$

Finally

$$\text{Set} \quad K_p = K_p/3$$

Treat all the values in the above formula as double precision.

In addition, the time of interpolation is delayed from the current time by separate delays $\delta F_{10.7}$ and δK_p for the daily and hourly values. Current time is given in modified Julian date.

$\delta F_{10.7} = 1.26 + 0.37\sin(H - 1.6057029)$, where H is obtained by subtracting the right ascension of the Sun from the right ascension of the satellite.

$\delta K_p = 0.1 + 0.2\cos^2(geomagnetic \quad latitude)$.

- **Barlier DTM78 (Drag Temperature Model)** [Barlier-1] The DTM78 model uses the same basic input values as the Jacchia-Bass model. It is a atmospheric density and temperature model based on satellite drag data and used data from two solar cycles in its development. It is not currently implemented in the operational program.

- **NSWCDD Exponential** [Smith] The NSWCDD model provides an analytic formula for the density function in the form of an exponential function of the satellite's height above the Earth. In addition there is a minimum height below which the model does not work and a maximum height above which the density is zero and in a neighborhood of which the model also does not work. Above a height of 2500Km the exponential model is not used and a straight line model is defined which takes the density values from the exponential model value at 2500Km down to zero. Thus the complete density model is given by

$$\begin{array}{ll} \rho = MaxDen & h < MinHt \\ \rho = e^{Ah-B-(Ch^2+Dh-E)^{1/2}} & MinHt \leq h < TopHt \\ \rho = TopDen + slope * (h - TopHt) & TopHt \leq h < MaxHt \\ \rho = MinDen & h \geq MaxHt \end{array} \qquad (11.100)$$

$$MinHt = 82Km \qquad MaxDen = 9277.22363091791522D+00Kg/m^3$$
$$MaxHt = 5499Km \qquad MinDen = 0.0D+00Kg/m^3$$
$$TopHt = 2500Km \qquad TopDen = .429133332397357233D-06Kg/m^3$$
$$slope = -0.143092141512956730D-09$$
$$A = +.13620D-01 \qquad B = -.83355D+01$$
$$C = +.10180D-03 \qquad D = +.10830D+01$$
$$E = +.89390D+02$$

11.6.2 Parameter Partials

The partial derivatives with respect to position, velocity, and the drag acceleration scaling parameter, C_d, are given by

$$\partial A_{drag}/\partial r = -(1/2)C_d\rho(s/m)|v_r|\Big[(1/\rho)v_r(\partial\rho/\partial r)^T - (I+\hat{v}_r\hat{v}_r^T)\Omega_{\vec{\omega}}\Big] \tag{11.101}$$

$$\partial A_{drag}/\partial r = -(1/2)C_d\rho(s/m)|v_r|\Big[(1/\rho)v_r\Big(\frac{\partial\rho}{\partial h}\frac{\partial h}{\partial r}\Big)^T - (I+\hat{v}_r\hat{v}_r^T)\Omega_{\vec{\omega}}\Big] \tag{11.102}$$

$$\partial A_{drag}/\partial \dot{r} = -(1/2)C_d\rho(s/m)|v_r|\Big[I+\hat{v}_r\hat{v}_r^T\Big] \tag{11.103}$$

$$\partial A_{drag}/\partial C_d = -(1/2)\rho(s/m)|v_r|v_r \tag{11.104}$$

$\partial\rho/\partial h$ is calculated by each of the three atmospheric density models, with the computation for the NSWCDD Exponential model given by

$$\partial\rho/\partial h = \rho\left[A - \frac{Ch+D/2}{\sqrt{Ch^2+Dh-E}}\right] \tag{11.105}$$

Height (h) Calculation

$r = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ Earth-fixed satellite position

$p = \sqrt{x^2 + y^2}$

$q = \tan(\phi)$ tangent of Geodetic Latitude

$f(q) = z/p + (a_e e^2/p)\left(q/\sqrt{1+(1-e^2)q^2}\right)$

$tol = .1D-10 \quad tol1 = .1D-02$

If $p < tol1$

$$h = |z| - a_e\sqrt{1-e^2} \qquad \partial h/\partial r = (0,\ 0,\ sign(z)) \tag{11.106}$$

Elseif $|z| < tol1$

$h = p - a_e \qquad \partial h/\partial r = (x/p,\ y/p,\ 0)$

Else Iterate for $q = \tan(\phi)$.

Endif

(Initialize)

$q_{old} = z/p$

(Start Iteration)

$q_{new} = f(q_{old})$

If $|q_{new} - q_{old}| \geq tol$

$q_{old} = q_{new}$

(Return to Start Iteration)

Else Calculate q, h, and $\partial h/\partial r$

$q = q_{new}$

$$h = \left[p + zq - a_e\sqrt{1+(1-e^2)q^2}\right] \Big/ \sqrt{1+q^2} \tag{11.107}$$

$$\partial h/\partial r = \left[1/\sqrt{1+q^2}\right] factor \tag{11.108}$$

where

$$factor = \partial p/\partial r + q\partial z/\partial r + \left[z - a_e(1-e^2)q\Big/\sqrt{1+(1-e^2)q^2} - hq\Big/\sqrt{1+q^2}\right]\partial q/\partial r$$

$$\partial p/\partial r = \begin{bmatrix} x/p \\ y/p \\ 0 \end{bmatrix} \qquad \partial z/\partial r = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\partial q/\partial r = \partial z/\partial r - q(\partial p/\partial r)\Big/\left(p - a_e e^2\Big/\left[1+(1-e^2)q^2\right]^{3/2}\right)$$

(End Iteration)

11.7 Solar Radiation Pressure

Acceleration due to solar radiation pressure is given by several models.

Satellite Radiation Models

Satellite	Model	Comment
Generic	Spherical	Generic Standard Model Description
GPS Block II A	Rock42	OCS Model (Rockwell, IBM, Aerospace)
GPS Block II A	T20	Aerospace (H. Fliegel, T. Gallini and E. Swift)
GPS Block II A	T20JPL	JPL (Bar-Sever 4/12/1997 Report)
GPS Block II A	T20JPL2	JPL (Bar-Sever 6/5/1998 Memo)
GPS Block II R	BLKIIR	OCS Model (CP-MCSSP-304B Appendix A)
GPS Block II R	T30	Aerospace (H. Fliegel and T. Gallini)
GPS Block II A	TJPLIIA	JPL (Bar-Sever ION 2003 Presentation)
GPS Block II R	TJPLIIR	JPL (Bar-Sever ION 2003 Presentation)
GPS Block II A	TJPLIIA2	JPL (Bar-Sever 12/2003 Memo)
GPS Block II R	TJPLIIR2	JPL (Bar-Sever 12/2003 Memo)

For those satellites using the spherical radiation model the force components can be represented in any frame. However, for the GPS satellites, the components of the force are given in the GPS local body frame. For these satellites there are currently two frames. One for Block II A type satellites and another for the Block II R satellites. In each case the acceleration vector, in the local body frame, will be represented by A^b.

If the local body frame column vectors are given by L_1, L_2, L_3 in the inertial frame then the radiation acceleration vector in the inertial frame, $A_{radiation} = A^i$ will be given by

$$A_{radiation} = A^i = LA^b(t) = \begin{bmatrix} L_1 & L_2 & L_3 \end{bmatrix} \begin{bmatrix} A_1^b \\ A_2^b \\ A_3^b \end{bmatrix} \tag{11.109}$$

11.7.1 Frames and Transformations

The transformation L, given below, will either convert from the Block II A frame or from the Block II R frame to the inertial frame. In each case the system is designed to point the navigation antennas toward the center of the Earth and to keep the solar panel axis perpendicular to the line of sight from the satellite to the Sun. In the Block II A case the $L_3 = z_b$ axis will point toward the center of the Earth, the $L_2 = y_b$ axis is along the solar panels and $L_1 = x_b$ completes a right-handed coordinate system. Block II R is similar in that L_3 is unchanged but L_2 and L_1 are reversed in direction from Block II A.

(GPSIIA) The GPS Body (XYZ) frame for Block II A.

$$\begin{gathered} L_1 = x_b = y_b \times z_b \quad L_2 = y_b = \widehat{z_b \times r_s} \quad L_3 = z_b = -\hat{r} \\ \text{If} \quad \sin(z_b, r_s) = |z_b \times r_s|/(|z_b||r_s|) < \varepsilon \\ \text{Then} \quad L_2 = \widehat{\dot{z}_b \times r_s} = \widehat{-\dot{r} \times r_s} \\ \varepsilon = \text{small number} \end{gathered} \tag{11.110}$$

(GPSIIRi) The GPS Body (XYZ) Ideal frame for Block II R.

$$\begin{gathered} L_1 = x_b = y_b \times z_b \quad L_2 = y_b = \widehat{r_s \times z_b} \quad L_3 = z_b = -\hat{r} \\ \text{If} \quad \sin(z_b, r_s) = |z_b \times r_s|/(|z_b||r_s|) < \varepsilon \\ \text{Then} \quad L_2 = \widehat{r_s \times \dot{z}_b} = \widehat{\dot{r} \times r_s} \end{gathered} \tag{11.111}$$

(GPSIIRβ) The GPS Body (XYZ) Low Beta frame for Block II R.

$$L_1 = L_2 \times L_3 \quad L_2 = -\widehat{r \times \dot{r}} \quad L_3 = -\hat{r} \tag{11.112}$$

The Ideal and Low Beta Block II R frames will be discussed below, in section 11.7.4, page 65.

11.7.2 Frame Partials

$$\Omega_v = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} \quad \text{for any vector } v. \tag{11.113}$$

(GPSIIA) The GPS Body (XYZ) frame for Block II A.

$$\begin{aligned} &\partial L_1/\partial r = \Omega_{L_2} \partial L_3/\partial r - \Omega_{L_3} \partial L_2/\partial r \\ &\partial L_1/\partial \dot{r} = 0 \\ &\partial L_2/\partial r = (I - (\widehat{\Omega_{r_s} L_3})(\widehat{\Omega_{r_s} L_3})^T)(\Omega_{r_s}/(|r||\Omega_{r_s} L_3|)) \\ &\partial L_2/\partial \dot{r} = 0 \\ &\partial L_3/\partial r = (L_3 L_3^T - I)/|r| \\ &\partial L_3/\partial \dot{r} = 0 \end{aligned} \tag{11.114}$$

(GPSIIRi) The GPS Body (XYZ) Ideal frame for Block II R.

$$\begin{aligned} &\partial L_1/\partial r = \Omega_{L_2} \partial L_3/\partial r - \Omega_{L_3} \partial L_2/\partial r \\ &\partial L_1/\partial \dot{r} = 0 \\ &\partial L_2/\partial r = -(I - (\widehat{\Omega_{r_s} L_3})(\widehat{\Omega_{r_s} L_3})^T)(\Omega_{r_s}/(|r||\Omega_{r_s} L_3|)) \\ &\partial L_2/\partial \dot{r} = 0 \\ &\partial L_3/\partial r = (L_3 L_3^T - I)/|r| \\ &\partial L_3/\partial \dot{r} = 0 \end{aligned} \tag{11.115}$$

(GPSIIRβ) The GPS Body (XYZ) Low Beta frame for Block II R.

$$
\begin{aligned}
&\partial L_1/\partial r = \Omega_{L_2}\partial L_3/\partial r - \Omega_{L_3}\partial L_2/\partial r \\
&\partial L_1/\partial \dot{r} = -\Omega_{L_3}\partial L_2/\partial \dot{r} \\
&\partial L_2/\partial r = -(L_2L_2^T - I)\Omega_{\dot{r}}/|\Omega_{\dot{r}} r| \\
&\partial L_2/\partial \dot{r} = -(I - L_2L_2^T)\Omega_r/|\Omega_r \dot{r}| \\
&\partial L_3/\partial r = -(I - L_3L_3^T)/|r| \\
&\partial L_3/\partial \dot{r} = 0
\end{aligned}
\tag{11.116}
$$

11.7.3 Parameter Partials

The partial derivatives with respect to position, velocity, and radiation parameters, Kr_1, Kr_2, and Kr_3 are given by

$$
\begin{aligned}
\partial A^i/\partial r &= (\partial L/\partial r)A^b + L(\partial A^b/\partial r) \\
\partial A^i/\partial \dot{r} &= (\partial L/\partial \dot{r})A^b + L(\partial A^b/\partial \dot{r}) \\
\partial A^i/\partial Kr &= (\partial A^i/\partial A^b)(\partial A^b/\partial Kr) = L(\partial A^b/\partial Kr) \\
\partial A^b/\partial r &= \partial A^b/\partial \dot{r} = 0
\end{aligned}
\tag{11.117}
$$

A^b and $\partial A^b/\partial Kr$ will be calculated for each model.

11.7.4 Radiation Acceleration Models

- Spherical Model- The spherical radiation model provides the acceleration formula below where there is only one Kr parameter. A^i will be computed directly as $L = I$ for the spherical model.

  $$A^i = shapeKr(s/m)cc(r - r_s)/|r - r_s|^3 \tag{11.118}$$

  shape is the shape function of 7.3, page 20, Kr is the radiation scaling parameter, s is the satellite cross-sectional area in square kilometers, m is the satellite mass in kilograms, and cc is the product of the solar constant, 10^6, and the square of the astronomical unit. The solar constant is in newtons per square meter and the astronomical unit is in kilometers. The solar constant can also be expressed as the mean solar flux at one astronomical unit divided by the speed of light.

  Partials are

  $$
  \begin{aligned}
  \partial A^i/\partial r &= shapeKr(s/m)cc(1/|r - r_s|^3)(I - 3\widehat{(r - r_s)}\widehat{(r - r_s)}^T) \\
  \partial A^i/\partial \dot{r} &= 0 \\
  \partial A^i/\partial Kr &= shape(s/m)cc(r - r_s)/|r - r_s|^3
  \end{aligned}
  \tag{11.119}
  $$

- Rock42 - OCS Official Block II A Model- [Moore] A^b for the official OCS Block II A model is computed in subroutine ROCK42. This code was formulated by the Rockwell International Corporation and coded by the IBM and Aerospace Corporations.

  Denote the accelerations provided by subroutine ROCK42 as $Rock42_x$, $Rock42_y$, and $Rock42_z$, where $Rock42_y = 0$, as this model does not calculate a component along the y_b axis. However, a y_b axis component has been identified by Aerospace Corporation and subsequently added to the model as a constant acceleration on the order of $10^{-12} km/sec^2$ with a scaling parameter Kr_2 of order 1. The total A^b acceleration is

$$A^b = \begin{bmatrix} Kr_1 Rock42_x + shape Kr_2 10^{-12}\cos(Kr_3) \\ shape Kr_2 10^{-12}\sin(Kr_3) + Rock42_y \\ Kr_1 Rock42_z \end{bmatrix} \tag{11.120}$$

$$\partial A^b/\partial Kr_1 = \begin{bmatrix} Rock42_x \\ 0 \\ Rock42_z \end{bmatrix} \tag{11.121}$$

$$\partial A^b/\partial Kr_2 = \begin{bmatrix} shape 10^{-12}\cos(Kr_3) \\ shape 10^{-12}\sin(Kr_3) \\ 0 \end{bmatrix} \tag{11.122}$$

$$\partial A^b/\partial Kr_3 = \begin{bmatrix} -shape Kr_2 10^{-12}\sin(Kr_3) \\ shape Kr_2 10^{-12}\cos(Kr_3) \\ 0 \end{bmatrix} \tag{11.123}$$

$$A^i = LA^b \tag{11.124}$$

$$\partial A^i/\partial Kr = L(\partial A^b/\partial Kr) \tag{11.125}$$

  In the above formula the *shape* function, section 7.3, page 20, is a part of the Rock42 acceleration computation but the y_b axis term is not considered to be part of the basic model and must be added in after the model acceleration is computed. The *shape* factor must then be applied to the y_b axis term as well.

  In addition the ROCK42 subroutine does not scale the acceleration to the actual distance of the satellite from the Sun. The subroutine calculates accelerations at one astronomical unit, *astro*, and scaling is carried out later.

  However, for the purpose of this presentation the scaling will be carried out inside the model subroutine, i.e., it is assumed that subroutine ROCK42 returns accelerations scaled to the actual satellite distance from the Sun.

  The assumption on scaling will apply to all radiation model descriptions.

  The scaling factor *scale* is

$$scale = (astro/|r_s - r|)^2 \tag{11.126}$$

- T20 - Aerospace Block II A Model- [Fliegel] The $T20_x$, $T20_y$, and $T20_z$ components are computed in subroutine T20 and are given by

$$
\begin{aligned}
&T20_x = shape \cdot scale \cdot (10^{-8}/m)(-8.96\sin(B) + 0.16\sin(3B) \\
&\qquad + 0.10\sin(5B) - 0.07\sin(7B)) \\
&T20_y = 0 \\
&T20_z = shape \cdot scale \cdot (10^{-8}/m)(-8.43\cos(B))
\end{aligned} \tag{11.127}
$$

where the scale factor of $10^{-8}/m$ converts from units of 10^{-5}newtons to kilometers per second squared. The angle B is the angle between the satellite-Sun direction and the satellite $L_3 = z_b$ axis, given in equation 11.110, page 64. B is in the range [0°, 180°].

Once the $T20$ values are computed they can replace the $Rock42$ values in equation 11.120, page 66 to provide the necessary inertial accelerations and partials. The accelerations and partials are given by

$$
A^b = \begin{bmatrix} Kr_1 T20_x + shape Kr_2 10^{-12}\cos(Kr_3) \\ shape Kr_2 10^{-12}\sin(Kr_3) + T20_y \\ Kr_1 T20_z \end{bmatrix} \tag{11.128}
$$

$$
A^i = LA^b \tag{11.129}
$$

$$
\partial A^b/\partial Kr_1 = \begin{bmatrix} T20_x \\ 0 \\ T20_z \end{bmatrix} \tag{11.130}
$$

$$
\partial A^b/\partial Kr_2 = \begin{bmatrix} shape 10^{-12}\cos(Kr_3) \\ shape 10^{-12}\sin(Kr_3) \\ 0 \end{bmatrix} \tag{11.131}
$$

$$
\partial A^b/\partial Kr_3 = \begin{bmatrix} -shape Kr_2 10^{-12}\sin(Kr_3) \\ shape Kr_2 10^{-12}\cos(Kr_3) \\ 0 \end{bmatrix} \tag{11.132}
$$

$$
\partial A^i/\partial Kr = L(\partial A^b/\partial Kr) \tag{11.133}
$$

- T30 - Aerospace Block II R Model- [Gallini] The $T30_x$, $T30_y$, and $T30_z$ components are computed in subroutine T30 and are given by

$$
\begin{aligned}
&T30_x = shape \cdot scale \cdot (10^{-8}/m)(+11.0\sin(B) \\
&\qquad + 0.2\sin(3B) - 0.2\sin(5B)) \\
&T30_y = 0 \\
&T30_z = shape \cdot scale \cdot (10^{-8}/m)(-11.3\cos(B) \\
&\qquad + 0.1\cos(3B) + 0.2\cos(5B))
\end{aligned} \tag{11.134}
$$

where the scale factor of $10^{-8}/m$ converts from units of 10^{-5}newtons to kilometers per second squared.

The $T30_x$ and $T30_y$ components must be reversed from the values calculated in the referenced paper as the x_b axis and the y_b axis for the Block II R body frame is reversed from those same axis directions in the Block II A body frame. See equation 11.111, page 64. The analysis in the referenced paper, above, assumed the same body axis structure for the $T30$ model as Block II A. The presentation above incorporates reversing these directions.

The $T30$ values could now replace the *Rock*42 values in equation 11.120, page 66, but in the case of Block II R satellites the function *shape* is not used with the y_b axis term.

Incorporating this change will provide the formula for the necessary inertial accelerations and partials. The result below for the T30 model is

$$A^b = \begin{bmatrix} Kr_1 T30_x + Kr_2 10^{-12}\cos(Kr_3) \\ Kr_2 10^{-12}\sin(Kr_3) + T30_y \\ Kr_1 T30_z \end{bmatrix} \tag{11.135}$$

$$A^i = LA^b \tag{11.136}$$

$$\partial A^b/\partial Kr_1 = \begin{bmatrix} T30_x \\ 0 \\ T30_z \end{bmatrix} \tag{11.137}$$

$$\partial A^b/\partial Kr_2 = \begin{bmatrix} 10^{-12}\cos(Kr_3) \\ 10^{-12}\sin(Kr_3) \\ 0 \end{bmatrix} \tag{11.138}$$

$$\partial A^b/\partial Kr_3 = \begin{bmatrix} -Kr_2 10^{-12}\sin(Kr_3) \\ Kr_2 10^{-12}\cos(Kr_3) \\ 0 \end{bmatrix} \tag{11.139}$$

$$\partial A^i/\partial Kr = L(\partial A^b/\partial Kr) \tag{11.140}$$

- BLKIIR - OCS Official Block II R Model- [Block II R], [Swift-2] The OCS Block II R model is a table look-up model, section 14.13, page 150, with entries in the table being body-fixed force values in newtons. The values are given in one degree increments of azimuth angle for a full range of values and also in one degree increments of elevation angle between negative four and positive four degrees. The table provides this variation as it was designed for a model using two modes for Block II R operation. One mode, the "Ideal mode" and the other, "Low-Beta mode". However, in practice only the "Ideal mode" has been used. In this mode only the zero degree elevation values are used and a linear interpolation is carried out to calculate the acceleration at a given azimuth angle. The description here however, is for the original model, including both "Ideal" and "Low-Beta" modes of operation. Taking $x_b = L_1$, $y_b = L_2$, and $z_b = L_3$ to be the Low-Beta frame of equation 11.112, page 64, at a given time, define

$$\begin{aligned} s_x &= \hat{r}_s^T x_b & \beta &= (180/\pi)\arcsin(-s_y) \\ s_y &= \hat{r}_s^T y_b & \alpha &= (180/\pi)\arctan(-s_x, -s_z) \\ s_z &= \hat{r}_s^T z_b & \alpha &= \alpha + 360 \quad \text{IF } \alpha < 0 \end{aligned} \tag{11.141}$$

s_{xyz} are the components of the unit Earth-Sun vector in the "Low-Beta" frame. β is the angle between the Satellite-Sun vector or the Earth-Sun vector and the orbit plane. It is positive if the Satellite-Sun vector forms an acute angle with the orbit normal (angular momentum) and negative otherwise. α is the orbit angle measured from the noon vector (projection of the Satellite-Sun vector on the orbit plane) to the satellite, positive in the direction of the orbit motion. At various times, such as an integration time or write time, α and β are computed and a decision must be made regarding which mode should be used for the satellite local body frame. Accelerations are then computed in this frame.

- **Transition Rules (Block II R Ideal/Low-Beta Mode)** β_{mode} will be known at the beginning of the last integration step, $last_{time}$. It will have to be determined at the end of the integration step (beginning of the next step), $current_{time}$, and at a write time that may have occurred since the end of the previous integration step, i.e., at a time that is greater than the start of the current step and less than or equal to the end of the current step. Let such a time be denoted as a $test_{time}$. Determine the value of $\beta_{mode}(test_{time})$, i.e., 'Ideal' or 'Low-Beta'. Define

$$\begin{aligned} \beta_{tran} &= 1.6\,\text{deg} && \text{(Low-Beta mode tolerance)} \\ \alpha_{lower} &= 75.0\,\text{deg} && \text{(transition window lower limit)} \\ \alpha_{upper} &= 105.0\,\text{deg} && \text{(transition window upper limit)} \end{aligned} \tag{11.142}$$

Using α and β above, at the $test_{time}$

$$\begin{aligned} \text{If} \quad \beta_{mode}(last_{time}) &= \text{'Ideal' and } |\beta| \le \beta_{tran} \\ \text{or} \quad \beta_{mode}(last_{time}) &= \text{'Low-Beta' and } |\beta| > \beta_{tran} \end{aligned} \tag{11.143}$$

Determine if α is in the transition window i.e.,

$$\alpha_{lower} < \alpha < \alpha_{upper} \tag{11.144}$$

If α is in this window then set

$$\beta_{mode}(test_{time}) = \text{ the opposite of } \beta_{mode}(last_{time}) \tag{11.145}$$

- **Exceptions**
  * 1. Don't transition between modes during the start routine. Determine which mode you should be in at epoch and keep that mode in effect during the start routine.
  * 2. When writing ephemeris values at a designated write time, do not transition between modes if a transition did not occur during the just terminated integration step. Recall item H of section 5, page 15, RUNNING PROCEDURE, that when an integration step terminates, the procedure is to check if the next write time occurred during the just past integration interval. If it did occur, then radiation acceleration values, among other values, may need to be written to the ephemeris file. Should this occur, then the proper frame, "Ideal" or "Low-Beta", must be used for the radiation calculation. If a check on mode transition

at the write time indicates that transition not only should occur, but can occur, i.e., α is within the transition window, then a check should be made of what happened at the integration time. If no transition occured at the integration time then no transition should occur at the write time. If transition did occur at the integration time then transition at the write time should follow the decision of the transition test above.

- **Force Table Interpolation** After the transition decision the azimuth, az, and elevation, el, angles are computed and used for linear interpolation in the force table.

$$
\begin{aligned}
&\textbf{'Ideal mode'}\\
&az = 270 + (180/\pi)\arcsin(s_z)\,\text{deg} \quad \text{(azimuth)}\\
&el = 0\,\text{deg} \quad \text{(elevation)}\\
&\textbf{'Low-Beta mode'}\\
&az = 180 + \alpha\,\text{deg} \quad \text{(azimuth)}\\
&el = -\beta\,\text{deg} \quad \text{(elevation)}
\end{aligned}
\tag{11.146}
$$

This interpolation yields the stf_x, stf_y, and stf_z force components, in newtons, at one astronomical unit. The resulting acceleration values in km/sec^2 are given by

$$
\begin{aligned}
BLKIIR_x &= scale \cdot (10^{-3}/m) shape \cdot stf_x\\
BLKIIR_y &= scale \cdot (10^{-3}/m) shape \cdot stf_y\\
BLKIIR_z &= scale \cdot (10^{-3}/m) shape \cdot stf_z
\end{aligned}
\tag{11.147}
$$

In addition, the last line of the table provides a constant radiation force due to heat, $Fumbra_x$, $Fumbra_y$, and $Fumbra_z$, in newtons. These values must be scaled to convert to accelerations in km/sec^2, and added into the $BLKIIR$ values above to get the total $BLKIIR$ accelerations. The $Fumbra$ values must be added without scaling by the shape factor, $shape$, as the heat component is active even during umbra.

$$
\begin{aligned}
BLKIIR_x &= scale \cdot (10^{-3}/m)(shape \cdot stf_x + Fumbra_x)\\
BLKIIR_y &= scale \cdot (10^{-3}/m)(shape \cdot stf_y + Fumbra_y)\\
BLKIIR_z &= scale \cdot (10^{-3}/m)(shape \cdot stf_z + Fumbra_z)
\end{aligned}
\tag{11.148}
$$

Once the $BLKIIR$, values are computed they can replace the $T30$ values in equation 11.135, page 68, where the L transformation will be from the "Ideal mode" or "Low-Beta mode" as decided in the transition discussion. There is a slight change in this model in that the $BLKIIR_y$ component is non-zero and is also scaled by Kr_1. This will yield inertial accelerations and partials, almost as before, given by

$$
A^b = \begin{bmatrix} Kr_1 BLKIIR_x + Kr_2 10^{-12}\cos(Kr_3)\\ Kr_2 10^{-12}\sin(Kr_3) + kr_1 BLKIIR_y\\ Kr_1 BLKIIR_z \end{bmatrix}
\tag{11.149}
$$

$$
A^i = LA^b \tag{11.150}
$$

$$\partial A^b/\partial Kr_1 = \begin{bmatrix} BLKIIR_x \\ BLKIIR_y \\ BLKIIR_z \end{bmatrix} \tag{11.151}$$

$$\partial A^b/\partial Kr_2 = \begin{bmatrix} 10^{-12}\cos(Kr_3) \\ 10^{-12}\sin(Kr_3) \\ 0 \end{bmatrix} \tag{11.152}$$

$$\partial A^b/\partial Kr_3 = \begin{bmatrix} -Kr_2 10^{-12}\sin(Kr_3) \\ Kr_2 10^{-12}\cos(Kr_3) \\ 0 \end{bmatrix} \tag{11.153}$$

$$\partial A^i/\partial Kr = L(\partial A^b/\partial Kr) \tag{11.154}$$

- T20JPL - JPL Block II A Model- [Bar-Sever-1] The JPL name for the model reported in [Bar-Sever-1] is $GSPM.II.97 + CY_1$. This model will be referred to here as the T20JPL model.

  The $T20JPL_x$, $T20JPL_y$, and $T20JPL_z$ components are computed in subroutine T20JPL and are provided by the following procedure which starts by computing values $T20JPL_u$, $T20JPL_v$, and $T20JPL_w$ in the UVW frame and then converting those values to the GPS XYZ body frame.

  The UVW frame has U as a unit vector pointing from the sun to the satellite, V is the same as the Y vector, of the XYZ frame, given in equation 11.110, page 64, and W completes a right handed system.

  The components in the UVW frame are

$$\begin{aligned}
T20JPL_u &= shape \cdot scale \cdot (10^{-8}/m)(CU_0 \\
&\quad + SU_1 \sin(B) + CU_1 \cos(B) \\
&\quad + SU_2 \sin(2B) + CU_2 \cos(2B)) \\
T20JPL_v &= shape(10^{-12}) CV_1 \cos(B) \\
CV_1 &= 0.1 + 0.5\sin(\beta) + 0.3/\sin(\beta) \quad |\beta| \geq 14^\circ \\
&= 0.1 + 0.5\sin(sign(\beta)14^\circ) + 0.3/\sin(sign(\beta)14^\circ) \quad |\beta| < 14^\circ \\
T20JPL_w &= shape \cdot scale \cdot (10^{-8}/m)(SW_1 \sin(B) \\
&\quad + CW_1 \cos(B) + SW_2 \sin(2B) + CW_2 \cos(2B))
\end{aligned} \tag{11.155}$$

where the coefficients are given by

$$\begin{aligned}
CU_0 &= +8.981090 & & \\
SU_1 &= +0.713423 & SW_1 &= -0.025771 \\
CU_1 &= -0.148668 & CW_1 &= +0.732856 \\
SU_2 &= +0.101430 & SW_2 &= -0.742966 \\
CU_2 &= -0.000274 & CW_2 &= -0.005984
\end{aligned} \tag{11.156}$$

Since the model expressions are given in the UVW frame and the standard GPS model works in the GPS XYZ body frame the values are converted to the standard GPS frame. This is carried out by

$$T_{uvw2xyz} = \begin{bmatrix} -\sin(B) & 0 & \cos(B) \\ 0 & 1 & 0 \\ -\cos(B) & 0 & -\sin(B) \end{bmatrix} \tag{11.157}$$

$$\begin{bmatrix} T20JPL_x \\ T20JPL_y \\ T20JPL_z \end{bmatrix} = T_{uvw2xyz} \begin{bmatrix} T20JPL_u \\ T20JPL_v \\ T20JPL_w \end{bmatrix} \tag{11.158}$$

The $T20JPL_{xyz}$ components are obtained numerically by computing the $T20JPL_{uvw}$ components and using $T_{uvw2xyz}$ to convert them. However, the xyz components could be obtained analytically by expanding the last equation to give

$$\begin{aligned} T20JPL_x &= shape \cdot scale \cdot (10^{-8}/m)(CX_0 \\ &\quad + SX_1 \sin(B) + CX_1 \cos(B) \\ &\quad + SX_2 \sin(2B) + CX_2 \cos(2B) \\ &\quad + SX_3 \sin(3B) + CX_3 \cos(3B)) \\ T20JPL_y &= shape 10^{-12} CY_1 \cos(B) \\ &= shape 10^{-12} CV_1 \cos(B) \\ T20JPL_z &= shape \cdot scale \cdot (10^{-8}/m)(CZ_0 \\ &\quad + SZ_1 \sin(B) + CZ_1 \cos(B) \\ &\quad + SZ_2 \sin(2B) + CZ_2 \cos(2B) \\ &\quad + SZ_3 \sin(3B) + CZ_3 \cos(3B)) \end{aligned} \tag{11.159}$$

$$\begin{aligned} CX_0 &= (CW_1 - SU_1)/2 &&= +0.0097165 \\ SX_1 &= (CU_2 + SW_2)/2 - CU_0 &&= -9.3527100 \\ CX_1 &= (CW_2 - SU_2)/2 &&= -0.0537070 \\ SX_2 &= (SW_1 - CU_1)/2 &&= +0.0614485 \\ CX_2 &= (SU_1 + CW_1)/2 &&= +0.7231395 \\ SX_3 &= (SW_2 - CU_2)/2 &&= -0.3713460 \\ CX_3 &= (SU_2 + CW_2)/2 &&= +0.0477230 \\ CY_1 &= CV_1 && \\ CZ_0 &= -(CU_1 + SW_1)/2 &&= +0.0872195 \\ SZ_1 &= +(CW_2 - SU_2)/2 &&= -0.0537070 \\ CZ_1 &= -(CU_2 + SW_2)/2 - CU_0 &&= -8.6094700 \\ SZ_2 &= -(SU_1 + CW_1)/2 &&= -0.7231395 \\ CZ_2 &= +(SW_1 - CU_1)/2 &&= +0.0614485 \\ SZ_3 &= -(SU_2 + CW_2)/2 &&= -0.0477230 \\ CZ_3 &= +(SW_2 - CU_2)/2 &&= -0.3713460 \end{aligned} \tag{11.160}$$

where again the scale factor of $10^{-8}/m$ converts from units of 10^{-5}newtons to kilometers per second squared. As with the other models once the JPL values are computed they also can replace the *Rock*42 values in equation 11.120, page 66, to provide the necessary inertial accelerations and partials. The accelerations and partials are given by

$$A^b = \begin{bmatrix} Kr_1 T20JPL_x + shapeKr_2 10^{-12}\cos(Kr_3) \\ shapeKr_2 10^{-12}\sin(Kr_3) + T20JPL_y \\ Kr_1 T20JPL_z \end{bmatrix} \tag{11.161}$$

$$A^i = LA^b \tag{11.162}$$

$$\partial A^b/\partial Kr_1 = \begin{bmatrix} T20JPL_x \\ 0 \\ T20JPL_z \end{bmatrix} \tag{11.163}$$

$$\partial A^b/\partial Kr_2 = \begin{bmatrix} shape10^{-12}\cos(Kr_3) \\ shape10^{-12}\sin(Kr_3) \\ 0 \end{bmatrix} \tag{11.164}$$

$$\partial A^b/\partial Kr_3 = \begin{bmatrix} -shapeKr_2 10^{-12}\sin(Kr_3) \\ shapeKr_2 10^{-12}\cos(Kr_3) \\ 0 \end{bmatrix} \tag{11.165}$$

$$\partial A^i/\partial Kr = L(\partial A^b/\partial Kr) \tag{11.166}$$

- T20JPL2 - JPL Block II A Model- [Bar-Sever-2] A variation on the above form of the JPL model, T20JPL, is documented in a memo of JPL/Yoaz Bar-Sever to Aerospace/Henry Fliegel dated June 5, 1998 and is implemented here in the T20JPL2 model. The JPL name for this model is GSPM_XYZ.1, where 1 refers to the version. This document implements the GSPM_XYZ.1 model as T20JPL2. Although GSPM_XYZ.1 is not mathematically the same as $GSPM.II.97 + CY_1$, in [Bar-Sever-1], it is reported to be functionally equivalent to it. The $T20JPL2_x$, $T20JPL2_y$, and $T20JPL2_z$ components of this variation are computed in subroutine T20JPL2 and are given by

$$\begin{aligned} T20JPL2_x &= shape \cdot scale \cdot (10^{-8}/m)(SX_1\sin(B) \\ &\quad + SX_2\sin(2B) + SX_3\sin(3B) \\ &\quad + SX_4\sin(4B) + SX_5\sin(5B) \\ &\quad + SX_6\sin(6B) + SX_7\sin(7B)) \\ T20JPL2_y &= shape10^{-12}CY_1\cos(B) \\ &= shape10^{-12}CV_1\cos(B) \\ T20JPL2_z &= shape \cdot scale \cdot (10^{-8}/m)CZ_1\cos(B) \end{aligned} \tag{11.167}$$

with coefficient values

$$\begin{aligned} SX_1 &= -8.79279000 & SX_5 &= +0.18732900 \\ SX_2 &= -0.00875225 & SX_6 &= -0.00146316 \\ SX_3 &= +0.05088520 & SX_7 &= +0.04967120 \\ SX_4 &= +0.01356980 & CZ_1 &= -8.43000000 \end{aligned} \tag{11.168}$$

As with the other models once the JPL values are computed they also can replace the *Rock*42 values in equation 11.120, page 66, to provide the necessary inertial accelerations and partials. The accelerations and partials are given by

$$A^b = \begin{bmatrix} Kr_1 T20JPL2_x + shapeKr_2 10^{-12}\cos(Kr_3) \\ shapeKr_2 10^{-12}\sin(Kr_3) + T20JPL2_y \\ Kr_1 T20JPL2_z \end{bmatrix} \tag{11.169}$$

$$A^i = LA^b \tag{11.170}$$

$$\partial A^b/\partial Kr_1 = \begin{bmatrix} T20JPL2_x \\ 0 \\ T20JPL2_z \end{bmatrix} \tag{11.171}$$

$$\partial A^b/\partial Kr_2 = \begin{bmatrix} shape 10^{-12}\cos(Kr_3) \\ shape 10^{-12}\sin(Kr_3) \\ 0 \end{bmatrix} \tag{11.172}$$

$$\partial A^b/\partial Kr_3 = \begin{bmatrix} -shapeKr_2 10^{-12}\sin(Kr_3) \\ shapeKr_2 10^{-12}\cos(Kr_3) \\ 0 \end{bmatrix} \tag{11.173}$$

$$\partial A^i/\partial Kr = L(\partial A^b/\partial Kr) \tag{11.174}$$

- TJPLIIA - JPL Block II A Model- [Bar-Sever-3] This model will be referred to here as the TJPLIIA model. The $TJPLIIA_x$, $TJPLIIA_y$, and $TJPLIIA_z$ components are computed in subroutine TJPLIIA and are provided by the following procedure which starts by computing the values $TJPLIIA_u$, $TJPLIIA_v$, and $TJPLIIA_w$ in the UVW frame and then converting those values to the GPS XYZ body frame. The UVW frame has U as a unit vector pointing from the sun to the satellite, V is the same as the Y vector, of the XYZ frame, and W completes a right handed system.

$$\begin{aligned}
TJPLIIA_u &= shape \cdot scale \cdot (10^{-8}/m)(CU_0 \\
&\quad + SU_1 \sin(B) + CU_1 \cos(B) \\
&\quad + SU_2 \sin(2B) + CU_2 \cos(2B)) \\
TJPLIIA_v &= shape \cdot (10^{-12})(CV_1 \cos(B) + CV_2 \cos(2B)) \\
CV_1 &= 0.083 + 0.556\sin(\beta) + 0.273/\sin(\beta) \quad |\beta| \geq 14^\circ \\
&= 0.083 + 0.556\sin(sign(\beta)14^\circ) + 0.273/\sin(sign(\beta)14^\circ) \quad |\beta| < 14^\circ \\
CV_2 &= 0.188 \\
TJPLIIA_w &= shape \cdot scale \cdot (10^{-8}/m)(SW_1 \sin(B) \\
&\quad + CW_1 \cos(B) + SW_2 \sin(2B) + CW_2 \cos(2B))
\end{aligned} \tag{11.175}$$

with coefficients given by

$$
\begin{aligned}
CU_0 &= +8.881000 \\
SU_1 &= +0.000000 \quad SW_1 = +0.011000 \\
CU_1 &= +0.067000 \quad CW_1 = -0.120000 \\
SU_2 &= -0.008000 \quad SW_2 = +0.000000 \\
CU_2 &= +0.000000 \quad CW_2 = +0.036000
\end{aligned}
\tag{11.176}
$$

Since the model expressions are given in the UVW frame and the standard GPS model works in the GPS XYZ body frame the values are converted to the standard GPS frame. This is carried out by

$$
T_{uvw2xyz} = \begin{bmatrix} -\sin(B) & 0 & \cos(B) \\ 0 & 1 & 0 \\ -\cos(B) & 0 & -\sin(B) \end{bmatrix} \tag{11.177}
$$

$$
\begin{bmatrix} TJPLIIA_x \\ TJPLIIA_y \\ TJPLIIA_z \end{bmatrix} = T_{uvw2xyz} \begin{bmatrix} TJPLIIA_u \\ TJPLIIA_v \\ TJPLIIA_w \end{bmatrix} \tag{11.178}
$$

In practice the xyz results are calculated numerically by converting the uvw results with the above transformation. However, one could expand the transformation formula analytically, as in the T20JPL model, obtaining

$$
\begin{aligned}
TJPLIIA_x = {} & shape \cdot scale \cdot (10^{-8}/m)(CX_0 \\
& + SX_1 \sin(B) + CX_1 \cos(B) \\
& + SX_2 \sin(2B) + CX_2 \cos(2B) \\
& + SX_3 \sin(3B) + CX_3 \cos(3B)) \\
TJPLIIA_y = {} & shape \cdot (10^{-12})(CV_1 \cos(B) + CV_2 \cos(2B)) \\
TJPLIIA_z = {} & shape \cdot scale \cdot (10^{-8}/m)(CZ_0 \\
& + SZ_1 \sin(B) + CZ_1 \cos(B) \\
& + SZ_2 \sin(2B) + CZ_2 \cos(2B) \\
& + SZ_3 \sin(3B) + CZ_3 \cos(3B))
\end{aligned}
\tag{11.179}
$$

where again the scale factor of $10^{-8}/m$ converts from units of 10^{-5}newtons to kilometers per second squared. The factor *scale* is given in equation 11.126, page 66 and scales the model from one astronomical unit to the actual distance of the satellite from the Sun.

As with the other models once the JPL values are computed they also can replace the *Rock*42 values in equation 11.120, page 66, to provide the necessary inertial accelerations and partials. The accelerations and partials are given by

$$
A^b = \begin{bmatrix} Kr_1 TJPLIIA_x + shapeKr_2 10^{-12} \cos(Kr_3) \\ shapeKr_2 10^{-12} \sin(Kr_3) + TJPLIIA_y \\ Kr_1 TJPLIIA_z \end{bmatrix} \tag{11.180}
$$

$$
A^i = LA^b \tag{11.181}
$$

$$\partial A^b/\partial Kr_1 = \begin{bmatrix} TJPLIIA_x \\ 0 \\ TJPLIIA_z \end{bmatrix} \tag{11.182}$$

$$\partial A^b/\partial Kr_2 = \begin{bmatrix} shape10^{-12}\cos(Kr_3) \\ shape10^{-12}\sin(Kr_3) \\ 0 \end{bmatrix} \tag{11.183}$$

$$\partial A^b/\partial Kr_3 = \begin{bmatrix} -shapeKr_2 10^{-12}\sin(Kr_3) \\ shapeKr_2 10^{-12}\cos(Kr_3) \\ 0 \end{bmatrix} \tag{11.184}$$

$$\partial A^i/\partial Kr = L(\partial A^b/\partial Kr) \tag{11.185}$$

- TJPLIIR - JPL Block II R Model- [Bar-Sever-3] This model will be referred to here as the TJPLIIR model. The $TJPLIIR_x$, $TJPLIIR_y$, and $TJPLIIR_z$ components are computed in subroutine TJPLIIR and are provided by the following procedure which starts by computing the values $TJPLIIR_u$, $TJPLIIR_v$, and $TJPLIIR_w$ in the UVW frame and then converting those values to the GPS XYZ body frame. The UVW frame has U as a unit vector pointing from the Sun to the satellite, V is the same as the Y vector, of the XYZ frame, and W completes a right handed system.

$$\begin{aligned}
TJPLIIR_u &= shape \cdot scale \cdot (10^{-8}/m)(CU_0 \\
&\quad + SU_1\sin(B) + CU_1\cos(B) \\
&\quad + SU_2\sin(2B) + CU_2\cos(2B)) \\
TJPLIIR_v &= shape \cdot (10^{-12})(CV_1\cos(B) + CV_2\cos(2B)) \\
CV_1 &= -0.008 + 0.181\sin(\beta) + 0.097/\sin(\beta) \quad |\beta| \geq 14^\circ \\
&= -0.008 + 0.181\sin(sign(\beta)14^\circ) + 0.097/\sin(sign(\beta)14^\circ) \quad |\beta| < 14^\circ \\
CV_2 &= 0.061 \\
TJPLIIR_w &= shape \cdot scale \cdot (10^{-8}/m)(SW_1\sin(B) \\
&\quad + CW_1\cos(B) + SW_2\sin(2B) + CW_2\cos(2B))
\end{aligned} \tag{11.186}$$

with coefficients given by

$$\begin{aligned}
CU_0 &= +11.190000 & & \\
SU_1 &= +0.000000 & SW_1 &= -0.060000 \\
CU_1 &= +0.553000 & CW_1 &= -0.167000 \\
SU_2 &= -0.243000 & SW_2 &= +0.000000 \\
CU_2 &= +0.000000 & CW_2 &= +0.025000
\end{aligned} \tag{11.187}$$

Since the model expressions are given in the UVW frame and the standard GPS model works in the GPS XYZ body frame the values are converted to the standard GPS frame. This is carried out by using

$$T_{uvw2xyz} = \begin{bmatrix} -\sin(B) & 0 & \cos(B) \\ 0 & 1 & 0 \\ -\cos(B) & 0 & -\sin(B) \end{bmatrix} \tag{11.188}$$

and the J transformation, used to reverse the x and y directions, discussed below.

$$J = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11.189}$$

The final transformation is

$$\begin{bmatrix} TJPLIIR_x \\ TJPLIIR_y \\ TJPLIIR_z \end{bmatrix} = J \cdot T_{uvw2xyz} \begin{bmatrix} TJPLIIR_u \\ TJPLIIR_v \\ TJPLIIR_w \end{bmatrix} \tag{11.190}$$

One could expand these formulas as in the T20JPL model obtaining

$$\begin{aligned} TJPLIIR_x &= -shape \cdot scale \cdot (10^{-8}/m)(CX_0 \\ &\quad + SX_1 \sin(B) + CX_1 \cos(B) \\ &\quad + SX_2 \sin(2B) + CX_2 \cos(2B) \\ &\quad + SX_3 \sin(3B) + CX_3 \cos(3B)) \\ TJPLIIR_y &= -shape \cdot (10^{-12})(CV_1 \cos(B) + CV_2 \cos(2B)) \\ TJPLIIR_z &= shape \cdot scale \cdot (10^{-8}/m)(CZ_0 \\ &\quad + SZ_1 \sin(B) + CZ_1 \cos(B) \\ &\quad + SZ_2 \sin(2B) + CZ_2 \cos(2B) \\ &\quad + SZ_3 \sin(3B) + CZ_3 \cos(3B)) \end{aligned} \tag{11.191}$$

where again the scale factor of $10^{-8}/m$ converts from units of 10^{-5}newtons to kilometers per second squared. The factor *scale* is given in equation 11.126, page 66 and scales the model from one astronomical unit to the actual distance of the satellite from the Sun.

The $TJPLIIR_x$ and $TJPLIIR_y$ components must be reversed from the values calculated in the reference as the x_b axis and the y_b axis for the Block II R body frame is reversed from those same axis directions in the Block II A body frame and the analysis in the reference assumed the same axis structure for the $TJPLIIR$ model as Block II A. The presentation above incorporates reversing these directions by using the J matrix.

As with the other models once the JPL values are computed they also can replace the $T30$ values in equation 11.135, page 68, to provide the necessary inertial accelerations and partials. These accelerations and partials are

$$A^b = \begin{bmatrix} Kr_1 TJPLIIR_x + Kr_2 10^{-12} \cos(Kr_3) \\ Kr_2 10^{-12} \sin(Kr_3) + TJPLIIR_y \\ Kr_1 TJPLIIR_z \end{bmatrix} \tag{11.192}$$

$$A^i = LA^b \tag{11.193}$$

$$\partial A^b/\partial Kr_1 = \begin{bmatrix} TJPLIIR_x \\ 0 \\ TJPLIIR_z \end{bmatrix} \tag{11.194}$$

$$\partial A^b/\partial Kr_2 = \begin{bmatrix} 10^{-12}\cos(Kr_3) \\ 10^{-12}\sin(Kr_3) \\ 0 \end{bmatrix} \tag{11.195}$$

$$\partial A^b/\partial Kr_3 = \begin{bmatrix} -Kr_2 10^{-12}\sin(Kr_3) \\ Kr_2 10^{-12}\cos(Kr_3) \\ 0 \end{bmatrix} \tag{11.196}$$

$$\partial A^i/\partial Kr = L(\partial A^b/\partial Kr) \tag{11.197}$$

- TJPLIIA2 - JPL Block II A Model- [Bar-Sever-4] A variation on the TJPLIIA model is a concurrently developed model that used the XYZ rather than the UVW frame.

$$\begin{aligned} TJPLIIA2_x &= shape \cdot scale \cdot (10^{-8}/m)(SX_1 \sin(B) \\ &\quad + SX_2 \sin(2B) + SX_3 \sin(3B) \\ &\quad + SX_5 \sin(5B) + SX_7 \sin(7B)) \\ TJPLIIA2_y &= shape \cdot scale \cdot (10^{-8}/m)(CY_1 \cos(B) \\ &\quad + CY_2 \cos(2B)) \\ CY_1 &= 0.009172D0 + 0.053905D0 \sin(\beta) \\ &\quad + 0.026486D0/\sin(\beta) \quad |\beta| \geq 14^\circ \\ &= 0.009172D0 + 0.053905D0 \sin(sign(\beta)14^\circ) \\ &\quad + 0.026486D0/\sin(sign(\beta)14^\circ) \quad |\beta| < 14^\circ \\ CY_2 &= 0.172949185827030D-01 \\ TJPLIIA2_z &= shape \cdot scale \cdot (10^{-8}/m)(CZ_1 \cos(B) \\ &\quad + CZ_3 \cos(3B) + CZ_5 \cos(5B)) \end{aligned} \tag{11.198}$$

Coefficient values are

$$\begin{aligned} SX_1 &= -0.898198494807990D+01 \\ SX_2 &= -0.219041333765820D-01 \\ SX_3 &= +0.150780795424309D-01 \\ SX_5 &= +0.104044278079801D+00 \\ SX_7 &= +0.378632812005425D-02 \\ CZ_1 &= -0.860440265825333D+01 \\ CZ_3 &= +0.157824163365571D-01 \\ CZ_5 &= +0.553169376634545D-01 \end{aligned} \tag{11.199}$$

The scale factor of $10^{-8}/m$ converts from units of 10^{-5}newtons to kilometers per second squared. The factor *scale* is given in equation 11.126, page 66 and scales the model

from one astronomical unit to the actual distance of the satellite from the Sun. Note that unlike the previous model, TJPLIIA, in the new XYZ model all three components are to be scaled by the same common scale factor.

As with the other models once the JPL values are computed they also can replace the *Rock*42 values in equation 11.120, page 66, to provide the necessary inertial accelerations and partials. The accelerations and partials are given by

$$A^b = \begin{bmatrix} Kr_1 TJPLIIA2_x + shapeKr_2 10^{-12}\cos(Kr_3) \\ shapeKr_2 10^{-12}\sin(Kr_3) + TJPLIIA2_y \\ Kr_1 TJPLIIA2_z \end{bmatrix} \tag{11.200}$$

$$A^i = LA^b \tag{11.201}$$

$$\partial A^b/\partial Kr_1 = \begin{bmatrix} TJPLIIA2_x \\ 0 \\ TJPLIIA2_z \end{bmatrix} \tag{11.202}$$

$$\partial A^b/\partial Kr_2 = \begin{bmatrix} shape 10^{-12}\cos(Kr_3) \\ shape 10^{-12}\sin(Kr_3) \\ 0 \end{bmatrix} \tag{11.203}$$

$$\partial A^b/\partial Kr_3 = \begin{bmatrix} -shapeKr_2 10^{-12}\sin(Kr_3) \\ shapeKr_2 10^{-12}\cos(Kr_3) \\ 0 \end{bmatrix} \tag{11.204}$$

$$\partial A^i/\partial Kr = L(\partial A^b/\partial Kr) \tag{11.205}$$

- TJPLIIR2 - JPL Block II A Model- [Bar-Sever-4] A variation on the TJPLIIR model is a concurrently developed model that used the XYZ rather than the UVW frame. The $TJPLIIR2_x$, $TJPLIIR2_y$, and $TJPLIIR2_z$ components of this variation are computed in subroutine TJPLIIR2 and are given by

$$\begin{aligned} TJPLIIR2_x = {} & shape \cdot scale \cdot (10^{-8}/m)(SX_1 \sin(B) \\ & + SX_2 \sin(2B) + SX_3 \sin(3B) \\ & + SX_5 \sin(5B) + SX_7 \sin(7B)) \end{aligned} \tag{11.206}$$

$$\begin{aligned} TJPLIIR2_y = {} & shape \cdot scale \cdot (10^{-8}/m)(CY_1 \cos(B) \\ & + CY_2 \cos(2B)) \end{aligned} \tag{11.207}$$

$$\begin{aligned} CY_1 &= 0.001008D0 - 0.019996D0 \sin(\beta) \\ & \quad - 0.010715D0/\sin(\beta) \quad |\beta| \geq 14^\circ \\ &= 0.001008D0 - 0.019996D0 \sin(sign(\beta)14^\circ) \\ & \quad - 0.010715D0/\sin(sign(\beta)14^\circ) \quad |\beta| < 14^\circ \\ CY_2 &= -0.670600000000000D-02 \end{aligned}$$

$$\begin{aligned} TJPLIIR2_z = {} & shape \cdot scale \cdot (10^{-8}/m)(CZ_1 \cos(B) \\ & + CZ_3 \cos(3B) + CZ_5 \cos(5B)) \end{aligned} \tag{11.208}$$

Coefficient values are

$$
\begin{aligned}
SX_1 &= +0.109309304684571D+02 \\
SX_2 &= +0.127915180288438D+00 \\
SX_3 &= +0.276746498256363D+00 \\
SX_5 &= -0.204514956516677D+00 \\
SX_7 &= +0.568029299280108D-01 \\
CZ_1 &= -0.116408244072158D+02 \\
CZ_3 &= +0.627382332521572D-01 \\
CZ_5 &= +0.674170250089597D-01
\end{aligned}
\tag{11.209}
$$

The scale factor of $10^{-8}/m$ converts from units of 10^{-5}newtons to kilometers per second squared. The factor *scale* is given in equation 11.126, page 66 and scales the model from one astronomical unit to the actual distance of the satellite from the Sun. Note that unlike the previous model, TJPLIIR, in the new XYZ model all three components are to be scaled by the same common scale factor. In addition, this model was developed in the Block II R frame and thus does not need to have the $TJPLIIR2_x$ and $TJPLIIR2_y$ axis values reversed in sign.

As with the other models once the JPL values are computed they also can replace the $T30$ values in equation 11.135, page 68, to provide the necessary inertial accelerations and partials. These accelerations and partials are

$$
A^b = \begin{bmatrix} Kr_1 TJPLIIR2_x + Kr_2 10^{-12}\cos(Kr_3) \\ Kr_2 10^{-12}\sin(Kr_3) + TJPLIIR2_y \\ Kr_1 TJPLIIR2_z \end{bmatrix} \tag{11.210}
$$

$$
A^i = LA^b \tag{11.211}
$$

$$
\partial A^b/\partial Kr_1 = \begin{bmatrix} TJPLIIR2_x \\ 0 \\ TJPLIIR2_z \end{bmatrix} \tag{11.212}
$$

$$
\partial A^b/\partial Kr_2 = \begin{bmatrix} 10^{-12}\cos(Kr_3) \\ 10^{-12}\sin(Kr_3) \\ 0 \end{bmatrix} \tag{11.213}
$$

$$
\partial A^b/\partial Kr_3 = \begin{bmatrix} -Kr_2 10^{-12}\sin(Kr_3) \\ Kr_2 10^{-12}\cos(Kr_3) \\ 0 \end{bmatrix} \tag{11.214}
$$

$$
\partial A^i/\partial Kr = L(\partial A^b/\partial Kr) \tag{11.215}
$$

12 TRANSFORMATIONS

12.1 Osculating to Classical Elements

Compute the classical elements from the osculating elements of equation 2.2, page 6.

$$\begin{array}{ll} e_1 = a & e_4 = i \\ e_2 = e\sin(\omega) & e_5 = l + \omega \\ e_3 = e\cos(\omega) & e_6 = \Omega \end{array} \tag{12.1}$$

The classical elements are rewritten here from equation 2.3, page 6.

$$\begin{array}{ll} a = \text{semimajor axis} & i = \text{inclination} \\ e = \text{eccentricity} & l = \text{mean anomaly} \\ \omega = \text{argument of perigee} & \Omega = \text{right ascension} \end{array} \tag{12.2}$$

a, i, Ω are known so only e, ω, l remain to be calculated.

$$\begin{aligned} e &= ((e\sin(\omega))^2 + (e\cos(\omega))^2)^{1/2} \\ \omega &= \tan^{-1}(e\sin(\omega)/e\cos(\omega)) \\ l &= (l + \omega) - \omega \end{aligned} \tag{12.3}$$

If $e\cos(\omega) < 0$ then add π to ω, and if $\omega < 0$ then add 2π to ω in the above.

12.2 Rectangular Coordinates to Classical Keplerian Elements

Compute the classical Keplerian elements

$$\begin{array}{ll} a = \text{semi major axis} & \Omega = \text{right ascension} \\ e = \text{eccentricity} & \omega = \text{argument of perigee} \\ i = \text{inclination} & l = \text{mean anomaly} \end{array} \tag{12.4}$$

from the Cartesian coordinates $(r, \dot{r}) = (x, y, z, \dot{x}, \dot{y}, \dot{z})$.

$$\begin{aligned} a &= \mu|r|/(2\mu - |r||\dot{r}|^2) \\ e &= (\mu a - |h|)/(\mu a) \\ i &= \tan^{-1}\left[(x^2+y^2)^{1/2}/(|h|^{1/2}z)\right] \qquad 0 \le i \le \pi \\ \Omega &= \tan^{-1}(-x/y) \qquad 0 \le \Omega \le 2\pi \\ u &= \tan^{-1}\left[z/(\sin(i)(x\cos(\Omega) + y\sin(\Omega)))\right] \\ v &= \tan^{-1}\left[|h|r^T\dot{r}/(|h|^2 - \mu|r|)\right] \\ \omega &= u - v \\ E &= z\tan^{-1}\left[(1-e)^{1/2}\sin(v)/((1+e)^{1/2}(1+\cos(v)))\right] \\ l &= E - e\sin(E) \end{aligned} \tag{12.5}$$

where μ is the Earth gravity constant, $h = r \times \dot{r}$ is the angular momentum, u is the argument of latitude, ν is the true anomaly, and E is the eccentric anomaly.

12.3 Osculating Elements to Rectangular Coordinates

Compute the Cartesian coordinates $(x, y, z, \dot{x}, \dot{y}, \dot{z})$ from the osculating elements of equation 2.2, page 6.

$$\begin{aligned} e_1 &= a & e_4 &= i \\ e_2 &= e\sin(\omega) & e_5 &= l + \omega \\ e_3 &= e\cos(\omega) & e_6 &= \Omega \end{aligned} \tag{12.6}$$

$$\begin{aligned} e &= ((e\sin(\omega))^2 + (e\cos(\omega))^2)^{1/2} \\ \omega &= \tan^{-1}(e\sin(\omega)/e\cos(\omega)) \\ l &= (l+\omega) - \omega \end{aligned} \tag{12.7}$$

If $e\cos(\omega) < 0$ then add π to ω, and if $\omega < 0$ then add 2π to ω in the above. Solve Kepler's equation, $l = E - e\sin(E)$, to obtain E (eccentric anomaly). Solve for E by setting $f(E) = E - e\sin(E) - l$ and using Newton's method. Then calculate

$$\begin{aligned} X_\omega &= a(\cos(E) - e) & \dot{X}_\omega &= -(\mu a)^{1/2}\sin(E)/|r| \\ Y_\omega &= a(1-e^2)^{1/2}\sin(E) & \dot{Y}_\omega &= (\mu p)^{1/2}\cos(E)/|r| \\ |r| &= a(1 - e\cos(E)) \end{aligned} \tag{12.8}$$

where μ is the Earth gravity constant. Compute $(r, \dot{r})$ by

$$r = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = T \begin{bmatrix} X_\omega \\ Y_\omega \\ O_\omega \end{bmatrix}$$

$$\dot{r} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = T \begin{bmatrix} \dot{X}_\omega \\ \dot{Y}_\omega \\ \dot{O}_\omega \end{bmatrix} \tag{12.9}$$

$$\begin{aligned} T &= R_z(-\Omega)R_x(-i)R_z(-w) \\ &= \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{bmatrix} \begin{bmatrix} \cos w & -\sin w & 0 \\ \sin w & \cos w & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{12.10}$$

12.4 Rectangular Coordinates to Osculating Elements

The energy integral is

$$a = 1/(2/|r| - |\dot{r}|^2/\mu) \tag{12.11}$$

where r is satellite position and μ is the Earth's gravity constant. Using the polar form of an ellipse gives

$$|r| = p/(1 + e\cos(v)) = a(1 - e\cos(E)) \tag{12.12}$$

where p is the semi-latus rectum, $|h|/\mu$, h is the angular momentum, v is the true anomaly, and E is the eccentric anomaly. From the above

$$\begin{array}{ll} |\dot{r}| = ae\dot{E}\sin(E) & e\cos(E) = 1 - |r|/a \\ e\cos(E) = 1 - |r|/a & e\sin(E) = |\dot{r}|/(a\dot{E}) \end{array}$$

Kepler's equation, $l = E - e\sin(E)$, can be used to compute the mean motion

$$\dot{l} = \text{mean motion} = \mu^{1/2}/a^{3/2} = \dot{E}(1 - e\cos(E)) \tag{12.13}$$

Differentiating Kepler's equation and using the expression for the mean motion gives

$$\dot{E} = \mu^{1/2}/(a^{1/2}|r|)$$

$$\mu_1 = e\sin(E) = |r||\dot{r}|/(a\mu)^{1/2}$$

$$\mu_2 = e\cos(E) = |r||\dot{r}|^2/\mu - 1$$

$$e = (\mu_1^2 + \mu_2^2)^{1/2} \tag{12.14}$$

$$i = \cos^{-1}(h_3/|h|) \tag{12.15}$$

Define

$$\widehat{h} = h/|h| \qquad \widehat{U} = r/|r| \qquad \widehat{V} = \widehat{h} \times \widehat{U}$$

$$\begin{bmatrix} \widehat{P} \\ \widehat{Q} \end{bmatrix} = \begin{bmatrix} \cos(v) & -\sin(v) \\ \sin(v) & \cos(v) \end{bmatrix} \begin{bmatrix} \widehat{U} \\ \widehat{V} \end{bmatrix}$$

$$N = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times h = \begin{bmatrix} -h_2 \\ h_1 \\ 0 \end{bmatrix}$$

$$\widehat{N} = \begin{bmatrix} \cos(\Omega) \\ \sin(\Omega) \\ 0 \end{bmatrix}$$

It is clear that $\widehat{P}$ points to perigee and $\widehat{N}$ points to the right ascension of the ascending node. The right ascension Ω can be calculated from

$$\Omega = \tan^{-1}(-h_1/h_2) \tag{12.16}$$

Define

$$\widehat{M} = \widehat{h} \times \widehat{N} \tag{12.17}$$

This makes $\widehat{M}$ and $\widehat{N}$ an orthonormal system in the plane of the Kepler motion giving

$$\begin{gathered}\cos(\omega) = \widehat{P}^T\widehat{N} \qquad \sin(\omega) = \widehat{P}^T\widehat{M} \\ \text{or} \\ e\cos(\omega) = e\widehat{P}^T\widehat{N} \qquad e\sin(\omega) = e\widehat{P}^T\widehat{M}\end{gathered} \tag{12.18}$$

This permits the argument of perigee to be computed by

$$\begin{aligned}\omega &= \tan^{-1}(\sin(\omega)/\cos(\omega)) \\ l+\omega &= l+\tan^{-1}(\sin(\omega)/\cos(\omega))\end{aligned} \tag{12.19}$$

The rates of change of ω and Ω are given by

$$\dot{\omega} = \left(\frac{\mu}{a^3}\right)^{1/2}\left(\frac{3C_{2,0}a_e^2(1-5\cos^2(i))}{4a^2(1-e^2)^2}\right) \tag{12.20}$$

$$\dot{\Omega} = \left(\frac{\mu}{a^3}\right)^{1/2}\left(\frac{3C_{2,0}a_e^2\cos(i)}{2a^2(1-e^2)^2}\right) \tag{12.21}$$

where a_e is the semi-major axis of the reference ellipsoid and $C_{2,0}$ is the coefficient of the second harmonic in the Earth's oblateness expression. Two other values use $C_{2,0}$ and are available for diagnostic printing. They are the anomalistic and nodal periods of the satellite. The anomalistic period, P_A, is the time between successive crossings of the moving perigee and the nodal period, P_N, is the time between successive crossing of the ascending node.

These values, both mean and osculating, are given below. The mean value uses mean or Brouwer elements and the osculating uses osculating elements in the calculation.

$$\bar{P}_A = 2\pi\left(\frac{a^3}{\mu}\right)^{1/2}\left(1-\frac{3C_{2,0}a_e^2(3\sin^2(i)-2)}{4a^2(1-e^2)^{3/2}}\right) \tag{12.22}$$

$$P_A = 2\pi\left(\frac{a^3}{\mu}\right)^{1/2}\left(1-\frac{3C_{2,0}a_e^2|h|^3(3\sin^2(i)|\widehat{r}\times\widehat{N}|^2-1)}{2a^2(\mu|r|(1-e^2))^3}\right) \tag{12.23}$$

The associated nodal periods can be obtained by adding the difference, $P_N - P_A$, to either of the above values, where mean or osculating elements respectively must be used to compute $P_N - P_A$.

$$P_N - P_A = 2\pi\left(\frac{a^3}{\mu}\right)^{1/2}\left(\frac{3C_{2,0}a_e^2(4-5\sin^2(i))}{4a^2(1-e^2)^{1/2}(1+e\cos(\omega))^2}\right) \tag{12.24}$$

12.5 Partials of Coordinates wrt Osculating Elements

Given the Cartesian coordinates $(r,\dot{r}) = (x,y,z,\dot{x},\dot{y},\dot{z})$ compute the partials of position and velocity with respect to osculating orbital elements.

$$\begin{aligned}\partial r/\partial e\sin(\omega) = -\Big[&\sin(\omega+E)+e\sin(\omega)\\ &-e\cos(\omega)e\sin(E)(1+(1-e^2)^{1/2})^{-1}\Big](1-e^2)^{-1}r\\ &+\Big[-(1+|r|a^{-1}(1-e^2)^{-1})\cos(\omega+E)\\ &+|r|a^{-1}(1-e^2)^{-1}e\cos(\omega)(e\cos(E)+(1-e^2)^{1/2})\\ &\cdot(1+(1-e^2)^{1/2})^{-1}\Big](a^3/\mu)^{1/2}\dot{r}\end{aligned} \tag{12.25}$$

$$\begin{aligned}\partial \dot{r}/\partial e\sin(\omega) = \Big[&a(1-e^2)|r|^{-1}\cos(\omega+E)-e\sin(E)\sin(\omega+E)\\ &+e\cos(\omega)(1-e^2\cos^2(E))(1+(1-e^2)^{1/2})^{-1}\Big]\\ &\cdot(\mu/a^3)^{1/2}(1-e^2)^{-1}(a^2/|r|)r\\ &+\Big[\sin(\omega+E)-e\cos(\omega)e\sin(E)(1+(1-e^2)^{1/2})^{-1}\Big](1-e^2)^{-1}\dot{r}\end{aligned} \tag{12.26}$$

$$\begin{aligned}\partial r/\partial e\cos(\omega) = -\Big[&\cos(\omega+E)+e\cos(\omega)\\ &-e\sin(\omega)e\sin(E)(1+(1-e^2)^{1/2})^{-1}\Big](1-e^2)^{-1}r\\ &+\Big[-(1+|r|a^{-1}(1-e^2)^{-1})\sin(\omega+E)\\ &-|r|a^{-1}(1-e^2)^{-1}e\sin(\omega)(e\cos(E)+(1-e^2)^{1/2})\\ &\cdot(1+(1-e^2)^{1/2})^{-1}\Big](a^3/\mu)^{1/2}\dot{r}\end{aligned} \tag{12.27}$$

$$\begin{aligned}\partial \dot{r}/\partial e\cos(\omega) = -\Big[&a(1-e^2)|r|^{-1}\sin(\omega+E)\\ &+e\sin(E)\cos(\omega+E)\\ &+e\sin(\omega)(1-e^2\cos^2(E))(1+(1-e^2)^{1/2})^{-1}\Big]\\ &\cdot(\mu/a^3)^{1/2}(1-e^2)^{-1}(a^2/|r|)r\\ &+\Big[\cos(\omega+E)+e\sin(\omega)e\sin(E)(1+(1-e^2)^{1/2})^{-1}\Big](1-e^2)^{-1}\dot{r}\end{aligned} \tag{12.28}$$

$$\partial r/\partial a = r/a \tag{12.29}$$

$$\partial \dot{r}/\partial a = -\dot{r}/(2a) \tag{12.30}$$

$$\partial r/\partial(l+\omega) = (a^3/\mu)^{1/2}\dot{r} \tag{12.31}$$

$$\partial \dot{r}/\partial(l+\omega) = -(\mu/a^3)^{1/2}(a/|r|)^3 r \tag{12.32}$$

$$\partial r/\partial \Omega = \begin{bmatrix} -y \\ x \\ 0 \end{bmatrix} \tag{12.33}$$

$$\partial \dot{r}/\partial \Omega = \begin{bmatrix} -\dot{y} \\ \dot{x} \\ 0 \end{bmatrix} \tag{12.34}$$

$$\partial r/\partial i = \begin{bmatrix} z\sin(\Omega) & \\ -z\cos(\Omega) & \\ y\cos(\Omega) & -x\sin(\Omega) \end{bmatrix} \tag{12.35}$$

$$\partial \dot{r}/\partial i = \begin{bmatrix} \dot{z}\sin(\Omega) & \\ -\dot{z}\cos(\Omega) & \\ \dot{y}\cos(\Omega) & -\dot{x}\sin(\Omega) \end{bmatrix} \tag{12.36}$$

12.6 Calendar to Julian Date

In going from calendar (*yr*, *day*, sec) to Modified Julian Date (*MJD*) or Julian Date (*JD*), or the reverse, the following formulas are good through the year 2099.

$$MJD = 365(yr-1900) + Int\left[\frac{(yr-1900-1)}{4}\right] \tag{12.37}$$

$$+ day + 15019 + \frac{\sec}{86400}$$

$$JD = MJD + 2400000.5 \tag{12.38}$$

$$yr = Int\left[\frac{(MJD-15019)}{365}\right] + 1900 \tag{12.39}$$

$$day = Int\Big[MJD - \Big(365(yr-1900) + Int\left[\frac{(yr-1900-1)}{4}\right]$$

$$+ 15019\Big)\Big] \tag{12.40}$$

$$\sec = 86400\Big(MJD - Int\big[MJD\big]\Big) \tag{12.41}$$

$day \leq 0$ set $yr = yr - 1$ and recompute the day.

The JD 2400000.5 is zero hours November 18, 1858. MJD is the number of days from that date and 15019 is the MJD of 1900 zero hours UT.

If the value for sec is given in UT1 then the resulting computation gives Julian Date (*JD*). If the value is in Terrestrial Dynamical Time, also called Terrestrial Time and Ephemeris Time, $TDT = TT = ET$, then the resulting computation gives Julian Ephemeris Date (*JED*).

Generally time will be known in a system such as (GPS) or (UTC) and a conversion must be made to obtain $(UT1)$ or (TDT). The necessary transformations for this conversion are given in the next section.

12.7 Time System Conversions

Transformation formulas are required to go between the time systems: GPS time, UTC time, Ephemeris Time (ET), Terrestrial Dynamical Time (TDT), International Atomic Time (TAI), Atomic Time $(A.1)$, Universal Time $(UT1)$, and Barycentric Dynamical Time (TDB).

International Atomic Time, TAI, is obtained from the combined input of many atomic clocks. It is the standard for the *International System* (SI) second.

Terrestrial Dynamical Time, TDT, is offset from TAI by 32.184 seconds but has the same rate. The purpose of the offset is to maintain continuity between TDT and ET. TDT replaced ET at the beginning of 1984.

Planetary ephemeris is given in, yet another time, Barycentric Dynamical Time, which differs from TDT by relativistic terms. TDT will be used for TDB in the work here.

$$\begin{aligned} TDB &= TDT + 0^s.001658\sin(g) + 0^s.000014\sin(2g) \\ g &= 357.53^\circ + 0.98560028^\circ(JD - J2000) \end{aligned} \tag{12.42}$$

where g is the mean anomaly of the Earth in it's orbit around the Sun.

Coordinated Universal Time, UTC, has the same rate as TAI but has occasional jumps of one second. These leap second jumps are placed into UTC, on occasion, in order to keep it within 0.7 seconds of Earth rotation time, $UT1$. They typically occur once per year and are tabulated below and on the leap second file at 14.3, page 128. It is $UT1-UTC$ which is calculated and distributed on a regular basis, e.g., by the United States Naval Observatory via *http://maia.usno.navy.mil* or *http://www.iers.org*, so that conversions to $UT1$ and TDT can be carried out.

$UT1$ time is needed for input to the calculation of Λ, Greenwich True Sidereal Time (GST), which subsequently allows for the calculation of the rotation transformation, B, secion 12.11.3, page 110.

TDT is needed to evaluate the JPL ephemeris file, section 14.9, page 137, in order to obtain the position of the Sun, Moon, and planets.

If *systemtime* is GPS or UTC time then $UT1$ and TDT can be computed by

On and after Jan. 1, 1972

$$\begin{aligned} (GPS) \quad UT1 &= (UT1-UTC)+(UTC-TAI) \\ &\quad +(TAI-GPS)+GPS \\ (UTC) \quad UT1 &= (UT1-UTC)+UTC \\ (GPS) \quad TDT &= (TDT-TAI)+(TAI-GPS)+GPS \\ (UTC) \quad TDT &= (TDT-TAI)+(TAI-UTC)+UTC \end{aligned} \tag{12.43}$$

Before Jan. 1, 1972

$$
\begin{aligned}
(GPS) \quad UT1 &= (UT1-UTC)+(UTC-A.1)+(A.1-TAI) \\
&\quad +(TAI-GPS)+GPS \\
(UTC) \quad UT1 &= (UT1-UTC)+UTC \\
(GPS) \quad TDT &= (TDT-A.1)+(A.1-TAI) \\
&\quad +(TAI-GPS)+GPS \\
(UTC) \quad TDT &= (TDT-A.1)+(A.1-UTC)+UTC
\end{aligned}
\tag{12.44}
$$

The values needed for the computations are obtained as follows with $UT1-UTC$ being provided as described above and in the next section, Earth Orientation Values.

$$
TDT-TAI=32.184 \quad TDT-A.1=32.150 \quad TAI-GPS=19 \tag{12.45}
$$

On and after Jan. 1, 1972 ($TAI-UTC$) is calculated by

$$
\begin{aligned}
TAI-UTC &= (A.1-UTC)-(A.1-TAI) \\
&= (10.0343817+N)-(.0343817) \\
&= 10.0000000+N
\end{aligned}
\tag{12.46}
$$

where N is the number of leap seconds since 1972 tabulated below.

Announced Steps (N) in UTC

MJD	yr	day	N	MJD	yr	day	N
41317	72	1	0	45516	83	182	12
41499	72	183	1	46247	85	182	13
41683	73	1	2	47161	88	1	14
42048	74	1	3	47892	90	1	15
42413	75	1	4	48257	91	1	16
42778	76	1	5	48804	92	183	17
43144	77	1	6	49169	93	182	18
43509	78	1	7	49534	94	182	19
43874	79	1	8	50083	96	1	20
44239	80	1	9	50630	97	182	21
44786	81	182	10	51179	99	1	22
45151	82	182	11				

Before Jan. 1, 1972 the calculation was given as

$$
(A.1-UTC)_i = a_{i1}+a_{i2}T+a_{i3}T^2 \tag{12.47}
$$

where $T=MJD-MJD_i+1$ and the MJD_i are taken from the Appendix, page 157, to satisfy

$$
MJD_i \le MJD < MJD_{i+1}
$$

12.8 Earth Orientation Values

Earth orientation coefficients are provided by NGA, and can be used to generate polar motion and $UT1 - UTC$ values as shown below. NGA coefficients represent $UT1 - UTC$ without zonal tide effects so zonal tide effects[Yoder] must be added to the $UT1 - UTC$ term after generating these values from the NGA coefficients.

The coefficients are obtained by NGA fitting USNO polar motion and $UT1 - UTC$ values using the NGA Coefficient Algorithm described by equation 12.49, page 90. For Δx and Δy, 435 days of data, primarily made up of USNO final values, are used in the fit. For $UT1 - UTC$, 32 days of USNO Bulletin A Rapid Service values are used and only the RI and RJ coefficients are solved for. The coefficients RK_1, RK_2, RL_1, RL_2 are set to zero and the remaining RK, RL coefficients are non-zero but fixed constants. Setting these values to zero implies that R_1, R_2 are also not needed. It is important that these first four values be zero and that the last two use nominal values of 500 since historically this part of the $UT1 - UTC$ coefficient formula was not used. In this approach no zonal tide contribution is included in the NGA coefficients for $UT1 - UTC$, as their effects were removed from the USNO values prior to the fitting process. See also equation (12.54).

Earth orientation values are calculated for several days using the coefficients. Then a zonal tide correction term, from the formula in section 12.8.1, page 92, is added to each daily $UT1 - UTC$ value.

These values are written to a temporary file, the Daily Polar Motion Values File. When Earth orientation values for a particular time are desired, quadratic interpolation is performed on the relevant daily values from the file. Finally, a diurnal/semi-diurnal tide correction can be applied, which currently is calculated with the Ray model. The Ray model [Ray], is an implementation of the sub-daily tide effect on Earth orientation as described in [IERS96].

At the point in the program where the Ray model is applied it has been evaluated at the current time line for the purpose of rotation at that time. The time is *systemtime*, which is generally GPS time. After the Ray model has been applied $UT1 - UTC$ must still be converted to $UT1 - systemtime$, which is generally $UT1 - GPS$. This conversion takes place by subtracting the term $sysutc = (systemtime - UTC)$, which is generally $GPS - UTC$. The result is that this value becomes $(UT1 - UTC) - (systemtime - UTC) = UT1 - systemtime$. This is generally $UT1 - GPS$. In this way the Greenwich Hour Angle can be evaluated by converting the GPS time of the current integration time line to the proper UT1 time.

$$\begin{aligned} UT1 &= systemtime + (UT1 - systemtime) \\ e.g., \ UT1 &= GPS + (UT1 - GPS) \\ UT1 - systemtime &= \Delta t_2 \text{ of section 12.11.3, page 110} \end{aligned} \tag{12.48}$$

When NGA coefficients are being used $GPS - UTC = (GPS - TAI) + (TAI - UTC)$ and $TAI - UTC$ is provided in the set of NGA coefficients. If the Daily Polar Motion Values File is input directly then the $TAI - UTC$ value is calculated from equation 12.46, page 88, and the leap second file at section 14.3, page 128.

NGA Coefficient Algorithm

$$\Delta x(t) = A + B(t - ta) + \sum_{j=1}^{2} C_j \sin(2\pi(t - ta)/P_j) + \sum_{j=1}^{2} D_j \cos(2\pi(t - ta)/P_j) \tag{12.49}$$

$$\Delta y(t) = E + F(t - ta) + \sum_{j=1}^{2} G_j \sin(2\pi(t - ta)/Q_j) + \sum_{j=1}^{2} H_j \cos(2\pi(t - ta)/Q_j) \tag{12.50}$$

$$UT1 - UTC = RI + RJ(t - tb) + \sum_{j=1}^{4} RK_j \sin(2\pi(t - tb)/R_j) + \sum_{j=1}^{4} RL_j \cos(2\pi(t - tb)/R_j) \tag{12.51}$$

- A, C_j, D_j, E, G_j, and D_j are in arcseconds, in the range plus/minus one.
- B and F are in arcseconds per day, in the range plus/minus one.
- RI, RK_j, and RL_j are in seconds of time, in the range plus/minus one. $RK_1 = RK_2 = RL_1 = RL_2 = 0.0$ and the remaining RK, RL values are non-zero but fixed constants. See the NGA Earth Orientation Data Record Format on page 91 and the example that follows.
- RJ is in seconds of time per day, in the range plus/minus one.
- P_j, Q_j, and R_j are in days, in the range 0 to 500, a value of zero is valid only if both corresponding coefficients are zero. $P_1 = 365.25$ $P_2 = 435$ $Q_1 = 365.25$ $Q_2 = 435$ $R_1 = R_2 = 500$ $R_3 = 365.25$ $R_4 = 182.625$ (435 day Chandler, 365.25 day annual, and 182.625 day semi-annual periods).
- t, ta, tb, and Effectivity Date are Modified Julian Dates in the range 44000 to 64000, and the times are in UTC. Program searches on generation date using the first set of values with date less than or equal to the input search date.
- Delta AT = TAI - UTC = 10 + Number of leap seconds since 1972. The range is plus/minus 99.

- The Bulletin Serial Number is a unique NGA assigned integer in the range 1 to 9999.

NGA Earth Orientation Coefficients

Record	Position	Format	Name	Record	Position	Format	Name
1	1	F10.2	ta	4	1	F10.6	RL1
1	11	F10.6	A	4	11	F10.6	RL2
1	21	F10.6	B	4	21	F10.6	RL3
1	31	F10.6	C1	4	31	F10.6	RL4
1	41	F10.6	C2	4	41	F9.4	R1
1	51	F10.6	D1	4	50	F9.4	R2
1	61	F10.6	D2	4	59	F9.4	R3
1	71	F6.2	P1	4	68	F9.4	R4
1	77	4X	Fill	4	77	4X	Fill
2	1	F6.2	P2	5	1	I4	Delta AT
2	7	F10.6	E	5	5	I5	Bulletin Serial No.
2	17	F10.6	F	5	10	I6	Effectivity Date
2	27	F10.6	G1	5	17	I7	Generation Date
2	37	F10.6	G2	5	35	58X	Fill
2	47	F10.6	H1				
2	57	F10.6	H2				
2	67	F6.2	Q1				
2	73	F6.2	Q2				
2	79	2X	Fill				
3	1	F10.2	tb				
3	11	F10.6	RI				
3	21	F10.6	RJ				
3	31	F10.6	RK1				
3	41	F10.6	RK2				
3	51	F10.6	RK3				
3	61	F10.6	RK4				
3	71	10X	Fill				

Sample Record of NGA Earth Orientation Coefficients

```
 51794.00   .061322   .000000  -.044814  -.052984   .124613  -.146247365.25
435.00   .333793   .000000  -.115951   .146040  -.043408  -.048951365.25435.00
 51909.00   .092296  -.000593   .000000   .000000  -.022000   .006000
  .000000   .000000   .012000  -.007000 500.0000 500.0000 365.2500 182.6250
 32  146 52231  52228                          2001 322 2001 319
 ta          A         B         C1        C2        D1        D2    P1
 P2      E         F         G1        G2        H1        H2    Q1    Q2
 tb          RI        RJ        RK1       RK2       RK3       RK4
   RL1       RL2       RL3       RL4   R1        R2        R3      R4
 DeltaAT SerialNo. EffDate GenDate EffDate(not used) GenDate(not used)
```

There is an option to use Earth orientation values from any other source, e.g., USNO or IERS, *http://hpiers.obspm.fr/eop-pc/*. All that is required is that they be placed on a file with the same format as the file containing the NGA generated Earth orientation values discussed above. Thus this file will either be internally generated if NGA coefficients are used or it will be attached as an external file if daily pole values themselves are used.

Daily Polar Motion Value File

yr	day	Δx(radians)	Δy(radians)	UT1-UTC(seconds)
F5.0	F5.0	D20.14	D20.14	D20.14
2000.	294.	-.18136879810308D-06	.12774840497236D-05	.15931000000000D+00
2000.	295.	-.18985303752249D-06	.12819443355898D-05	.15842900000000D+00
2000.	296.	-.19693131726669D-06	.12867439910328D-05	.15734300000000D+00
2000.	297.	-.20386415290656D-06	.12933374570959D-05	.15601100000000D+00
2000.	298.	-.21011824939287D-06	.13010944759937D-05	.15454500000000D+00
2000.	299.	-.21491790483586D-06	.13087545321552D-05	.15303300000000D+00

12.8.1 Diurnal, Semi-Diurnal, and Zonal Tide Corrections

Daily and sub-daily tidal variations in both the Earth's rotation and polar motion are provided by Ray *et al.* [Ray] . Periodic zonal tide variations in UT1 due to tidal deformation of the polar moment of inertia were derived by Yoder *et al.* [Yoder] including the tidal deformation of the Earth with a decoupled core. These include variations from 5 days to 18.6 years. This work is documented in Chapter 8 of the IERS 96 Conventions [IERS96].

Semi-Diurnal and Diurnal Tides

The formulas from equation (12.52), below, due to Ray [Ray], provide for the corrections in diurnal (daily) and semi-diurnal (subdaily) tidal variations for both polar motion and the Earth's rotation. Values without the effect of the tide can be corrected by adding in the value of δx, δy and $\delta UT1$.

$$
\begin{aligned}
\delta x(10^{-3} arcseconds) &= \sum_{i=1}^{8} F_i \sin\xi_i + G_i \cos\xi_i \\
\delta y(10^{-3} arcseconds) &= \sum_{i=1}^{8} H_i \sin\xi_i + K_i \cos\xi_i \\
\delta UT1(10^{-3} seconds) &= \sum_{i=1}^{8} D_i \sin\xi_i + E_i \cos\xi_i \\
\xi_i(radians) &= mod(2\pi/S \sum_{j=1}^{6} c_{ij}\gamma_j, 2\pi) + \phi_i
\end{aligned}
\tag{12.52}
$$

The value $S(1296000)$ converts seconds of arc to radians and the γ_j are defined below. The c_{ij} are integer multipliers of the γ_j (l, l', F, D, Ω, θ) for the i^{th} tide (row i of the Diurnal and Subdiurnal Tide Terms for Polar Motion and UT1), i.e., the $c_{i1}, c_{i2}, \ldots, c_{i6}$ are the integer values in columns 2-7 of row i. ϕ_i is the phase in radians and is the entry in column 8 of row i. The period is in hours. The units are 10^{-3} arcseconds for δx and δy and 10^{-3} seconds for $\delta UT1$. Values for δx and δy are converted to radians and $\delta UT1$ is converted to seconds for use in the program. The γ_i values, in arcseconds, are computed above where each value is moded with 1296000 to remove full cycles. Fundamental Arguments computed here, in the previous section on Semi-Diurnal and Diurnal Tides, and in a later section on Nutation, could all use the same formula but the program subroutines were implemented at different times, and used code from different sources. Thus each implementation of the Fundamental Arguments is somewhat different.

Fundamental Arguments (arcseconds)

$$
\begin{aligned}
\gamma_1(l) &= \text{Mean anomaly of the Moon} &&= l_0 + l_1 t + l_2 t^2 + l_3 t^3 + l_4 t^4 \\
\gamma_2(l') &= \text{Mean anomaly of the Sun} &&= l'_0 + l'_1 t + l'_2 t^2 + l'_3 t^3 + l'_4 t^4 \\
\gamma_3(F) &= L - \Omega &&= F_0 + F_1 t + F_2 t^2 + F_3 t^3 + F_4 t^4 \\
\gamma_4(D) &= \text{Mean elongation (Moon from Sun)} &&= D_0 + D_1 t + D_2 t^2 + D_3 t^3 + D_4 t^4 \\
\gamma_5(\Omega) &= \text{Longitude of the Moon's mean node} &&= \Omega_0 + \Omega_1 t + \Omega_2 t^2 + \Omega_3 t^3 + \Omega_4 t^4 \\
\gamma_6(\theta) &= GreenwichMeanSiderealTime &&= \theta_0 + (36525.0S + \theta_1)t + \theta_2 t^2 \\
t &= (MJD - MJD_{J2000})/36525.0 &&\quad + \theta_3 t^3 + \theta_4 t^4 \\
L &= \text{Mean longitude of the Moon} && \\
&\text{Mod the } \gamma \text{ values with } S &&
\end{aligned}
\tag{12.53}
$$

Coefficients for Fundamental Arguments J2000

l_0	485868.249036	D_0	1072260.703692
l_1	1717915923.2178	D_1	1602961601.2090
l_2	31.8792	D_2	-6.3706
l_3	.051635	D_3	.006593
l_4	-.00024470	D_4	-.00003169
l'_0	1287104.793048	Ω_0	450160.398036
l'_1	129596581.0481	Ω_1	-6962890.5431
l'_2	-.5532	Ω_2	7.4722
l'_3	.000136	Ω_3	.007702
l'_4	-.00001149	Ω_4	-.00005939
F_0	335779.5262320	θ_0	361658.22615
F_1	1739527262.8478	θ_1	129602772.19299
F_2	-12.7512	θ_2	1.39656
F_3	-.001037	θ_3	-.000093
F_4	.00000417	θ_4	.000

Diurnal and Subdiurnal Tide Terms for Polar Motion and UT1

									δx		δy	
Tide	l	l'	F	D	Ω	θ	ϕ	Period	F	G	H	K
Q_1	-1	0	-2	0	-2	1	$-\pi/2$	26.868	-0.026	0.006	-0.006	-0.026
O_1	0	0	-2	0	-2	1	$-\pi/2$	25.819	-0.133	0.049	-0.049	-0.133
P_1	0	0	-2	2	-2	1	$-\pi/2$	24.066	-0.050	0.025	-0.025	-0.050
K_1	0	0	0	0	0	1	$\pi/2$	23.935	-0.152	0.078	-0.078	-0.152
N_2	-1	0	-2	0	-2	2	0	12.658	-0.057	-0.013	0.011	0.033
M_2	0	0	-2	0	-2	2	0	12.421	-0.330	-0.028	0.037	0.196
S_2	0	0	-2	2	-2	2	0	12.000	-0.145	0.064	0.059	0.087
K_2	0	0	0	0	0	2	0	11.967	-0.036	0.017	0.018	0.022

										$\delta UT1$
Tide	l	l'	F	D	Ω	θ	ϕ	Period	D	E
Q_1	-1	0	-2	0	-2	1	$-\pi/2$	26.868	0.0245	0.0503
O_1	0	0	-2	0	-2	1	$-\pi/2$	25.819	0.1210	0.1605
P_1	0	0	-2	2	-2	1	$-\pi/2$	24.066	0.0286	0.0516
K_1	0	0	0	0	0	1	$\pi/2$	23.935	0.0864	0.1771
N_2	-1	0	-2	0	-2	2	0	12.658	-0.0380	-0.0154
M_2	0	0	-2	0	-2	2	0	12.421	-0.1617	-0.0720
S_2	0	0	-2	2	-2	2	0	12.000	-0.0759	-0.0004
K_2	0	0	0	0	0	2	0	11.967	-0.0196	-0.0038

Zonal Tides

The zonal tide terms on page 96 provide corrections for the zonal tide variations in $UT1 - UTC$ with periods between five days and 18.6 years. Values without the effect of the tide can be corrected by adding $\delta UT1$, $\delta\Delta$, and $\delta\omega$ correction values from equation (12.55). Program options include making zonal tide corrections with $N = 0$, 41, or 62 terms. These terms correspond to no correction, corrections with periods of less than 35 days, and for greater than 35 days. Corrections with $N \neq 0$ are made, only to $\delta UT1$, when using NGA polar motion coefficients to generate polar motion values. This is because NGA removes the effect of zonal tides from USNO values before generating coefficients.

The six historically unused coefficients, from the NGA Coefficient Algorithm, could be replaced with values obtained from a least square fit to the zonal tide formula. The fitting function would be that portion of the NGA coefficient formula involving the six unused coefficients RK_1, RK_2, RL_1, RL_2, R_1, and R_2. R_1 and R_2 could be set to 27.56 days and 13.66 days respectively, the periods with the largest amplitude, among the first 41 terms of the zonal tide formula, page 96.

Approximate Zonal Tide Formula

$$UT1 - UTC = \sum_{j=1}^{2} RK_j \sin(2\pi(t-tb)/R_j) + \sum_{j=1}^{2} RL_j \cos(2\pi(t-tb)/R_j) \qquad (12.54)$$

In this way one could evaluate the $UT1 - UTC$ equation, from the NGA Coefficient Algorithm, 12.51, page 90, in the normal way, and include an approximation to the zonal tide correction formula.

The full correction formula with correction terms $\delta UT1$, $\delta\Delta$ (Length of Day), and $\delta\omega$ is given by

$$\begin{aligned}
\delta UT1(10^{-4}sec) &= \sum_{i=1}^{N} A_i \sin\xi_i \\
\delta\Delta(10^{-5}sec) &= \sum_{i=1}^{N} A_i' \cos\xi_i \\
\delta\omega(10^{-14}radians/sec) &= \sum_{i=1}^{N} A_i'' \cos\xi_i \\
\xi_i(radians) &= mod(2\pi/S\sum_{j=1}^{5} c_{ij}\gamma_j, 2\pi)
\end{aligned} \tag{12.55}$$

The value $S(1296000)$ converts seconds of arc to radians. The c_{ij} are integer multipliers of the γ_j (l, l', F, D, Ω) for the i^{th} tide (row i of the Zonal Tide Terms), i.e., the $c_{i1}, c_{i2}, \ldots, c_{i5}$ are the integer values in columns 2-6, row i of the listing on page 96. The period is in days.

Fundamental Arguments (arcseconds)

$$\begin{aligned}
\gamma_1(l) &= \text{Mean anomaly of the Moon} &&= l_0 + l_1 t \\
\gamma_2(l') &= \text{Mean anomaly of the Sun} &&= l_0' + l_1' t \\
\gamma_3(F) &= L - \Omega &&= F_0 + F_1 t \\
\gamma_4(D) &= \text{Mean elongation (Moon from Sun)} &&= D_0 + D_1 t \\
\gamma_5(\Omega) &= \text{Longitude of the Moon's mean node} &&= \Omega_0 + \Omega_1 t \\
t &= (MJD - MJD_{J2000})/36525.0 \\
L &= \text{Mean longitude of the Moon.} \\
&\text{Mod the } \gamma \text{ values with } S
\end{aligned} \tag{12.56}$$

Coefficients for Fundamental Arguments J2000

l_0	485856.000	D_0	1072260.000
l_1	1717915929.570	D_1	1602961586.010
l_0'	1287108.000	Ω_0	450144.000
l_1'	129596544.000	Ω_1	-6962921.460
F_0	335772.000		
F_1	1739527231.500		

Zonal Tide Terms

N	l	l'	F	D	Ω	Period	$\delta UT1$ A	$\delta\Delta$ A'	$\delta\omega$ A"
1	1	0	2	2	2	5.64	-0.024	0.26	-0.22
2	2	0	2	0	1	6.85	-0.040	0.37	-0.31
3	2	0	2	0	2	6.86	-0.099	0.90	-0.76
4	0	0	2	2	1	7.09	-0.051	0.45	-0.38
5	0	0	2	2	2	7.10	-0.123	1.09	-0.92
6	1	0	2	0	0	9.11	-0.039	0.27	-0.22
7	1	0	2	0	1	9.12	-0.411	2.83	-2.39
8	1	0	2	0	2	9.13	-0.993	6.83	-5.76
9	3	0	0	0	0	9.18	-0.018	0.12	-0.10
10	-1	0	2	2	1	9.54	-0.082	0.54	-0.45
11	-1	0	2	2	2	9.56	-0.197	1.30	-1.10
12	1	0	0	2	0	9.61	-0.076	0.50	-0.42
13	2	0	2	-2	2	12.81	0.022	-0.11	0.09
14	0	1	2	0	2	13.17	0.025	-0.12	0.10
15	0	0	2	0	0	13.61	-0.299	1.38	-1.17
16	0	0	2	0	1	13.63	-3.208	14.79	-12.48
17	0	0	2	0	2	13.66	-7.757	35.68	-30.11
18	2	0	0	0	-1	13.75	0.022	-0.10	0.08
19	2	0	0	0	0	13.78	-0.338	1.54	-1.30
20	2	0	0	0	1	13.81	0.018	-0.08	0.07
21	0	-1	2	0	2	14.19	-0.024	0.11	-0.09
22	0	0	0	2	-1	14.73	0.047	-0.20	0.17
23	0	0	0	2	0	14.77	-0.734	3.12	-2.64
24	0	0	0	2	1	14.80	-0.053	0.22	-0.19
25	0	-1	0	2	0	15.39	-0.051	0.21	-0.17
26	1	0	2	-2	1	23.86	0.050	-0.13	0.11
27	1	0	2	-2	2	23.94	0.101	-0.26	0.22
28	1	1	0	0	0	25.62	0.039	-0.10	0.08
29	-1	0	2	0	0	26.88	0.047	-0.11	0.09
30	-1	0	2	0	1	26.98	0.177	-0.41	0.35
31	-1	0	2	0	2	27.09	0.435	-1.01	0.85
32	1	0	0	0	-1	27.44	0.534	-1.22	1.03
33	1	0	0	0	0	27.56	-8.261	18.84	-15.90
34	1	0	0	0	1	27.67	0.544	-1.24	1.04
35	0	0	0	1	0	29.53	0.047	-0.10	0.08
36	1	-1	0	0	0	29.80	-0.055	0.12	-0.10
37	-1	0	0	2	-1	31.66	0.118	-0.23	0.20
38	-1	0	0	2	0	31.81	-1.824	3.60	-3.04
39	-1	0	0	2	1	31.96	0.132	-0.26	0.22
40	1	0	-2	2	-1	32.61	0.018	-0.03	0.03

Zonal Tide Terms

N	l	l'	F	D	Ω	Period	$\delta UT1$ A	$\delta\Delta$ A'	$\delta\omega$ A"
41	-1	-1	0	2	0	34.85	-0.086	0.15	-0.13
42	0	2	2	-2	2	91.31	-0.057	0.04	-0.03
43	0	1	2	-2	1	119.61	0.033	-0.02	0.01
44	0	1	2	-2	2	121.75	-1.885	0.97	-0.82
45	0	0	2	-2	0	173.31	0.251	-0.09	0.08
46	0	0	2	-2	1	177.84	1.170	-0.41	0.35
47	0	0	2	-2	2	182.62	-48.247	16.60	-14.01
48	0	2	0	0	0	182.63	-0.194	0.07	-0.06
49	2	0	0	-2	-1	199.84	0.049	-0.02	0.01
50	2	0	0	-2	0	205.89	-0.547	0.17	-0.14
51	2	0	0	-2	1	212.32	0.037	-0.01	0.01
52	0	-1	2	-2	1	346.60	-0.045	0.01	-0.01
53	0	1	0	0	-1	346.64	0.092	-0.02	0.01
54	0	-1	2	-2	2	365.22	0.828	-0.14	0.12
55	0	1	0	0	0	365.26	-15.359	2.64	-2.23
56	0	1	0	0	1	386.00	-0.138	0.02	-0.02
57	1	0	0	-1	0	411.78	0.035	-0.01	0.00
58	2	0	-2	0	0	1095.17	-0.137	-0.01	0.01
59	-2	0	2	0	1	1305.47	0.422	-0.02	0.02
60	-1	1	0	1	0	3232.85	0.040	0.00	0.00
61	0	0	0	0	2	3399.18	7.900	0.15	-0.12
62	0	0	0	0	1	6790.36	-1617.268	-14.95	12.62

12.9 Lagrange Interpolation

The Lagrange interpolation procedure of order n, for a function f, comes from constructing an order n polynomial function y from $n+1$ function values $f(t_i)$ of f, such that the function values agree with the polynomial values at each of the nodes t_i. The construction is carried out by setting

$$y(t) = \sum_{i=0}^{n} L_i(t) f(t_i) \tag{12.57}$$

and finding polynomials L_i, such that the above expression is an identity when y is itself a polynomial.

$$L_i(t) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \left(\frac{t - t_j}{t_i - t_j} \right) \tag{12.58}$$

$$\dot{y}(t) = \sum_{i=0}^{n} \dot{L}_i(t) f(t_i) \tag{12.59}$$

$$\ddot{y}(t) = \sum_{i=0}^{n} \ddot{L}_i(t) f(t_i) \tag{12.60}$$

$$\dot{L}_i(t) = L_i(t) \sum_{\substack{j=0 \\ j \neq i}}^{n} \left(\frac{1}{(t - t_j)} \right) \tag{12.61}$$

$$\ddot{L}_i(t) = L_i(t) \left\{ \left[\sum_{\substack{j=0 \\ j \neq i}}^{n} \left(\frac{1}{(t - t_j)} \right) \right]^2 - \sum_{\substack{j=0 \\ j \neq i}}^{n} \left(\frac{1}{(t - t_j)} \right)^2 \right\} \tag{12.62}$$

$n = 7$ for satellite ephemeris and partial derivatives

$n = 5$ for Sun/Moon ephemeris and nutation angles

$n = 2$ for polar motion and UT1-UTC type corrections

12.10 Canonical to Standard Partial Derivatives

Each partial derivative $\partial r / \partial p$ has a start time and an end time which gives the domain associated with the parameter p. The parameter is nonzero during this domain. Partial derivatives obtained as a result of the integration process are initialized at epoch and integrated over the entire trajectory. Consequently they do not represent a partial derivative which was initialized at the start of the domain of p. These canonical partial derivatives must be converted to the correctly initialized partials by taking into account their domain. This procedure is documented in [O'Toole].

Parameters p can be either orbit parameters such as position and velocity or orbital elements, at some epoch t_s, or non-orbit parameters such as drag, thrust, or radiation pressure scaling.

If p is a non-orbit parameter with start time t_s and end time t_e then the transformation is given by

$$\psi_p(t) = \left\{ \begin{array}{cc} 0 & t \le t_s \\ \psi_{\tilde{p}}(t) - \psi_{\tilde{e}}(t)\psi_{\tilde{e}}^{-1}(t_s)\psi_{\tilde{p}}(t_s) & t_s < t \le t_e \\ \psi_{\tilde{e}}(t)\left[\psi_{\tilde{e}}^{-1}(t_e)\psi_{\tilde{p}}(t_e) - \psi_{\tilde{e}}^{-1}(t_s)\psi_{\tilde{p}}(t_s)\right] & t_e < t \end{array} \right\} \tag{12.63}$$

where ψ_p is $\partial r/\partial p$, the correct partial, with initial value of zero at the start time t_s. $\psi_{\tilde{p}}$ is $\partial r/\partial \tilde{p}$, the canonical partial derivative obtained from integrating equation 2.5, page 6, with initial value of zero at epoch T_0. $\psi_{\tilde{e}}$ is the canonical set of partial derivatives, $\partial r/\partial e$, representing partials of r, at time t, with respect to the initial orbital elements, e, at epoch T_0. These partials were obtained by integrating equation 2.4, page 6 with initial conditions $T(t_s)$ where $t_s = 0$. The value r represents position/velocity $(r, \dot{r})$ and e represents position/velocity $(r, \dot{r})$ or orbital elements $e = (e_1, e_2, ..., e_6)$ at some epoch.

If p is the orbit parameter set e, at some epoch t_s, then the necessary transformation is given by

$$\psi_e(t) = \psi_{\tilde{e}}(t)\psi_{\tilde{e}}^{-1}(t_s)T(t_s) \tag{12.64}$$

$$T(t_s) = \left\{ \begin{array}{cc} \partial r(t_s)/\partial e(t_s) & e = \text{ orbital elements} \\ I & e = \text{ position/velocity} \end{array} \right\} \tag{12.65}$$

where now ψ_e is the set of partials of r, at time t, with respect to the initial orbital elements, e, at the new epoch t_s. Had these values been obtained by integration then they would be the solution to equation 2.4, page 6 with initial conditions $T(t_s)$ where $t_s \neq 0$.

These results are based on the observation that two different solutions to the non-homogeneous equation 2.5, page 6 differ by a linear combination of the independent solutions to the homogeneous equation 2.4, page 6. These independent solutions are the columns of the matrix $\psi_{\tilde{e}}$.

12.11 Inertial Earth-Fixed Rotation (ABCD)

Documentation for A, B, C and D of *ABCD* is contained in [Darnell], [IERS96], [Kaplan], [Lieske-1], and [Seidelmann]. There are some deviations from [IERS96] where the formulation follows the other references. The convention here is that the Earth-fixed coordinates $\bar{x}$ are obtained from the inertial coordinates x by applying the *ABCD* matrix as shown. The starting frame for this transformation is the frame defined by the mean pole and equinox of J2000.0. The realizations of this pole and equinox have respective uncertainties of 50 mas and 80 mas [Fricke],[Feissel], due to errors in the FK5. This pole is slightly offset from the ICRS pole, the CEP. These celestial pole offsets are regularly published by the IERS and are available at *http://hpiers.obspm.fr/eop-pc/*. See also *http://aa.usno.navy.mil/faq/docs/ICRS_doc.html.*

The epoch J2000.0 is JD 2451545.0 TDT. By using the conversions from section 12.7, page 87, it is also 2000 January 1 12h TDT, or 2000 January 1 11:59:27.816 TAI, or 2000 January 1 11:58:55.816 UTC.

$$\bar{x} = \begin{pmatrix} e' \\ f' \\ g' \end{pmatrix} = ABCDx \tag{12.66}$$

12.11.1 Precession - D

The D matrix [Lieske-2] accounts for lunisolar and planetary precession. It transforms the J2000.0 pole and equinox to the mean equator and equinox of the current epoch. This procedure follows Option 1, Chapter 5, from [IERS96].

Time is calculated as TDT in the computation of JED. TDT is approximately equal to TDB and system time is converted to TDT. These calculations are done in section 12.7, page 87. The expression for D satisfies the reflexive property in that replacing z by $-\zeta$, ζ by $-z$, and θ by $-\theta$ gives D^{-1}.

$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} = R_3(-z)R_2(\theta)R_3(-\zeta) \tag{12.67}$$

where the basic rotations, $R_i(angle)$, are counterclockwise rotations, of the given $angle$, about the x_i axis.

The d_{ij} are given by

$$\begin{aligned}
d_{11} &= \cos(z)\cos(\theta)\cos(\zeta) - \sin(z)\sin(\zeta) \\
d_{12} &= -\cos(z)\cos(\theta)\sin(\zeta) - \sin(z)\cos(\zeta) \\
d_{13} &= -\cos(z)\sin(\theta) \\
d_{21} &= \sin(z)\cos(\theta)\cos(\zeta) + \cos(z)\sin(\zeta) \\
d_{22} &= -\sin(z)\cos(\theta)\sin(\zeta) + \cos(z)\cos(\zeta) \\
d_{23} &= -\sin(z)\sin(\theta) \\
d_{31} &= \sin(\theta)\cos(\zeta) \\
d_{32} &= -\sin(\theta)\sin(\zeta) \\
d_{33} &= \cos(\theta)
\end{aligned} \tag{12.68}$$

where the angles ζ, z, and θ are given by

$$\begin{aligned}
\zeta &= (2306.''2181 + 1.''39656T - 0.''000139T^2)t \\
&\quad + (0.''30188 - 0.''000344T)t^2 + 0.''017998t^3 \\
z &= (2306.''2181 + 1''.39656T - 0.''000139T^2)t \\
&\quad + (1.''09468 + 0.''000066T)t^2 + 0.''018203t^3 \\
&= \zeta + (0.''7928 + 0.''00041T)t^2 + 0.''000205t^3 \\
\theta &= (2004.''3109 - 0.''8533T - 0.''000217T^2)t \\
&\quad + (-0.''42665 - 0.''000217T)t^2 - 0.''041833t^3
\end{aligned} \tag{12.69}$$

$$T = \frac{\left(JED(E_f) - JED(E_o)\right)}{J}$$

$$t = \frac{\left(JED(E_d) - JED(E_f)\right)}{J}$$

$$J = 36525$$

$$JED(E_o) = J2000 = 2451545.0TDT$$

$JED(E_f)$ is the Julian Ephemeris Date of the arbitrary beginning epoch. $JED(E_o)$ is $J2000.0$, the J2000 epoch of precession. $JED(E_d)$ is the arbitrary ending epoch of precession. Intervals of time are given in Julian centuries of $J = 36525$ days. Generally the arbitrary beginning inertial reference epoch E_f will be $J2000$ and $JED(E_f) = JED(E_o)$. The formulas however, allow for an inertial system epoch E_f other than $J2000$. The parameters are based on the 1980 revision of the IAU(1976) System of Astronomical Constants [Seidelmann][Lieske-1][Lieske-2] and the Fifth Fundamental Catalog (FK5)[Fricke].

For the purpose of processing historical data the equations for the Besselian based system will be maintained. These equations are based on the Besselian reference epoch of 1950.0 and intervals of time are given in Tropical centuries of $J = 36524.2198781$ days.

The formulas are given below for the angles ζ, z, and, θ.

$$\begin{aligned}
\zeta &= (2305.''64601783 + 1.''396T - 0.''0T^2)t \\
&\quad + (0.''302 - 0.''0T)t^2 + 0.''018t^3 \\
z &= \zeta + (0.''791 + 0.''0T)t^2 + 0.''0t^3 \\
\theta &= (2003.''8289891 - 0.''853T - 0.''0T^2)t \\
&\quad + (-0.''426 - 0.''0T)t^2 - 0.''042t^3
\end{aligned} \tag{12.70}$$

$$T = \frac{(JED(E_f) - JED(E_o))}{J}$$

$$t = \frac{(JED(Ed) - JED(E_f))}{J}$$

$$J = 36524.2198781$$

$$JED(E_o) = B1950.0 = 2433282.42345905$$

12.11.2 Nutation - C

The nutation transformation transforms the mean equator and equinox frame of the current epoch to the true equator and equinox of that time. The pole for this frame is viewed as the Celestial Ephemeris Pole (CEP) of the epoch. This alignment will be more precise if the grayed out part of the equations below, using the celestial pole offsets, were to be implemented.

If the J2000 system is being used the Fundamental Arguments of J2000 and the Wahr Nutation Series are used to calculate this transformation, [Seidelmann] and [Wahr].

The B1950.0 system is maintained for historical processing. In this system the calculation uses the Fundamental Arguments of B1950.0 and the Woolard's Coefficients.

System time is converted to TDT using 12.7, page 87 for the purpose of this calculation. The expression for C satisfies the reflexive property in that replacing $\bar{\varepsilon}$ by ε, ε by $\bar{\varepsilon}$, and $\Delta\psi$ by $-\Delta\psi$ gives C^{-1}.

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = R_1(-\varepsilon)R_3(-\Delta\psi)R_1(\bar{\varepsilon}) \tag{12.71}$$

$$\begin{aligned} c_{11} &= \cos(\Delta\psi) \\ c_{12} &= -\sin(\Delta\psi)\cos(\bar{\varepsilon}) \\ c_{13} &= -\sin(\Delta\psi)\sin(\bar{\varepsilon}) \end{aligned} \tag{12.72}$$

$$\begin{aligned} c_{21} &= \cos(\varepsilon)\sin(\Delta\psi) \\ c_{22} &= \cos(\varepsilon)\cos(\Delta\psi)\cos(\bar{\varepsilon}) + \sin(\varepsilon)\sin(\bar{\varepsilon}) \\ c_{23} &= \cos(\varepsilon)\cos(\Delta\psi)\sin(\bar{\varepsilon}) - \sin(\varepsilon)\cos(\bar{\varepsilon}) \end{aligned} \tag{12.73}$$

$$\begin{aligned} c_{31} &= \sin(\varepsilon)\sin(\Delta\psi) \\ c_{32} &= \sin(\varepsilon)\cos(\Delta\psi)\cos(\bar{\varepsilon}) - \cos(\varepsilon)\sin(\bar{\varepsilon}) \\ c_{33} &= \sin(\varepsilon)\cos(\Delta\psi)\sin(\bar{\varepsilon}) + \cos(\varepsilon)\cos(\bar{\varepsilon}) \end{aligned} \tag{12.74}$$

$$\begin{aligned} \bar{\varepsilon} &= \text{mean obliquity (degrees)} \\ &= E_0 + E_1 t + E_2 t^2 + E_3 t^3 \end{aligned} \tag{12.75}$$

$$\begin{aligned} \varepsilon &= \text{true obliquity (degrees)} \\ &= \bar{\varepsilon} + \Delta\varepsilon/3600 \end{aligned} \tag{12.76}$$

$\Delta\varepsilon$ = nutation in obliquity (arcseconds)

$\Delta\psi$ = nutation in longitude (arcseconds) measured from the true vernal equinox in the ecliptic plane.

$$S = 1296000 \text{ seconds of arc to radians}$$

$$\begin{aligned} \Delta\alpha_i(arcseconds) &= X(1,i)l + X(2,i)l' + X(3,i)F \\ &\quad + X(4,i)D + X(5,i)\Omega \end{aligned} \tag{12.77}$$

$$\Delta\alpha_i(radians) = mod(2\pi/S\Delta\alpha_i, 2\pi)$$

$$\Delta\alpha_i(radians) = 2\pi/Smod(\Delta\alpha_i, S) \quad \text{(alternate form)}$$

$$\begin{aligned} \Delta\psi &= (10^{-4})\sum_{i=1}^{N}(X(6,i) + X(7,i)t)\sin(\Delta\alpha_i) \\ &\quad + \text{dpsi} \end{aligned} \tag{12.78}$$

$$\begin{aligned} \Delta\varepsilon &= (10^{-4})\sum_{i=1}^{N}(X(8,i) + X(9,i)t)\cos(\Delta\alpha_i) \\ &\quad + \text{deps} \end{aligned} \tag{12.79}$$

where the correction terms, dpsi and deps, are provided in the IERS Bulletins as "celestial pole offsets", and they have been grayed out here to indicate that they have not been implemented in the operational OrbGgen program.

The above computations can be completed by using the table information below, starting with the computation of the Fundamental Arguments and then computing the $\Delta\alpha_i$. Mod $\Delta\alpha_i$ with $S = 1296000$ to remove full cycles and multiply by $2\pi/S$ to convert from seconds of arc to radians.

Fundamental Arguments (arcseconds)

$$\begin{aligned}
l &= \text{Mean anomaly of the Moon} &&= l_0 + l_1 t + l_2 t^2 + l_3 t^3 \\
& &&\quad + (llt - [llt])S \\
l' &= \text{Mean anomaly of the Sun} &&= l_0' + l_1' t + l_2' t^2 + l_3' t^3 \\
& &&\quad + (l'l't - \left[l'l't\right])S \\
F &= L - \Omega &&= F_0 + F_1 t + F_2 t^2 + F_3 t^3 \\
& &&\quad + (FFt - [FFt])S \\
D &= \text{Mean elongation (Moon from Sun)} &&= D_0 + D_1 t + D_2 t^2 + D_3 t^3 \\
& &&\quad + (DDt - [DDt])S \\
\Omega &= \text{Longitude of the Moon's mean node} &&= \Omega_0 + \Omega_1 t + \Omega_2 t^2 + \Omega_3 t^3 \\
& &&\quad - (\Omega\Omega t - [\Omega\Omega t])S
\end{aligned} \tag{12.80}$$

L = Mean longitude of the Moon

Mod all values with S

Coefficients for Fundamental Arguments B1950.0

S	1296000.000		
ll	1325.000	DD	1236.000
l_0	485866.52179	D_0	1072261.66679
l_1	715923.1253	D_1	1105600.860
l_2	33.22454	D_2	-5.1496
l_3	.0518	D_3	.0068
$l'l'$	99.000	$\Omega\Omega$	5.000
l'_0	1287091.548	Ω_0	450156.048
l'_1	1292577.984	Ω_1	-482896.246
l'_2	-.5724	Ω_2	7.504
l'_3	-.0108	Ω_3	.008
FF	1342.000	E_0	23.439280808
F_0	335782.17879	E_1	-.013014269
F_1	295267.4164	E_2	-.000000131
F_2	-11.5636	E_3	.0000005036
F_3	-.0012		

Coefficients for Fundamental Arguments J2000

S	1296000.000		
ll	1325.000	DD	1236.000
l_0	485866.733	D_0	1072261.307
l_1	715922.633	D_1	1105601.328
l_2	31.310	D_2	-6.891
l_3	.064	D_3	.019
$l'l'$	99.000	$\Omega\Omega$	5.000
l'_0	1287099.804	Ω_0	450160.280
l'_1	1292581.224	Ω_1	-482890.539
l'_2	-.5770	Ω_2	7.455
l'_3	-.012	Ω_3	.008
FF	1342.000	E_0	23.4392911111
F_0	335778.877	E_1	-.0130041667
F_1	295263.137	E_2	-.0000001639
F_2	-13.257	E_3	.0000005036
F_3	.011		

Multiples of Arguments and Coefficients
Woolard's Coefficients N = 69
Updated from B1950.0 to Epoch J2000

X(1,:)	X(2,:)	X(3,:)	X(4,:)	X(5,:)	X(6,:)	X(7,:)	X(8,:)	X(9,:)
0.0	0.0	0.0	0.0	1.0	-172500.7	-173.7	92109.1	9.1
0.0	0.0	2.0	-2.0	2.0	-12730.3	-1.3	5519.1	-2.9
0.0	0.0	2.0	0.0	2.0	-2037.2	-0.2	883.5	-0.5
0.0	0.0	0.0	0.0	2.0	2088.2	0.2	-903.6	0.4
0.0	1.0	0.0	0.0	0.0	1257.9	-3.1	0.0	0.0
1.0	0.0	0.0	0.0	0.0	675.1	0.1	0.0	0.0
0.0	1.0	2.0	-2.0	2.0	-495.8	1.2	215.4	-0.6
0.0	0.0	2.0	0.0	1.0	-342.4	-0.4	183.0	0.0
1.0	0.0	2.0	0.0	2.0	-261.0	0.0	112.9	-0.1
0.0	-1.0	2.0	-2.0	2.0	213.5	-0.5	-92.7	0.3
1.0	0.0	0.0	-2.0	0.0	-149.0	0.0	0.0	0.0
0.0	0.0	2.0	-2.0	1.0	124.1	0.1	-66.0	0.0
-1.0	0.0	2.0	0.0	2.0	114.0	0.0	-50.0	0.0
1.0	0.0	0.0	0.0	1.0	58.0	0.0	-31.0	0.0
0.0	0.0	0.0	2.0	0.0	60.0	0.0	0.0	0.0
-1.0	0.0	2.0	2.0	2.0	-52.0	0.0	22.0	0.0
-1.0	0.0	0.0	0.0	1.0	-57.0	0.0	30.0	0.0
1.0	0.0	2.0	0.0	1.0	-44.0	0.0	23.0	0.0
2.0	0.0	0.0	-2.0	0.0	45.0	0.0	0.0	0.0
-2.0	0.0	2.0	0.0	1.0	45.0	0.0	-24.0	0.0
0.0	0.0	2.0	2.0	2.0	-32.0	0.0	14.0	0.0
2.0	0.0	2.0	0.0	2.0	-26.0	0.0	11.0	0.0
2.0	0.0	0.0	0.0	0.0	28.0	0.0	0.0	0.0
1.0	0.0	2.0	-2.0	2.0	26.0	0.0	-11.0	0.0
0.0	0.0	2.0	0.0	0.0	25.0	0.0	0.0	0.0
0.0	0.0	2.0	-2.0	0.0	-21.0	0.0	0.0	0.0
-1.0	0.0	2.0	0.0	1.0	19.0	0.0	-10.0	0.0
0.0	2.0	0.0	0.0	0.0	15.9	-0.1	0.0	0.0
0.0	2.0	2.0	-2.0	2.0	-14.9	0.1	7.0	0.0
-1.0	0.0	0.0	2.0	1.0	14.0	0.0	-7.0	0.0
0.0	1.0	0.0	0.0	1.0	-15.0	0.0	8.0	0.0
1.0	0.0	0.0	-2.0	1.0	-13.0	0.0	7.0	0.0
0.0	-1.0	0.0	0.0	1.0	-10.0	0.0	5.0	0.0
2.0	0.0	-2.0	0.0	0.0	10.0	0.0	0.0	0.0
-1.0	0.0	2.0	2.0	1.0	-9.0	0.0	5.0	0.0
1.0	0.0	2.0	2.0	2.0	-6.0	0.0	3.0	0.0
0.0	-1.0	2.0	0.0	2.0	-6.0	0.0	3.0	0.0
0.0	0.0	2.0	2.0	1.0	-5.0	0.0	3.0	0.0
1.0	1.0	0.0	-2.0	0.0	-7.0	0.0	0.0	0.0
0.0	1.0	2.0	0.0	2.0	7.0	0.0	-3.0	0.0

Multiples of Arguments and Coefficients
Woolard's Coefficients N = 69
Updated from B1950.0 to Epoch J2000

X(1,:)	X(2,:)	X(3,:)	X(4,:)	X(5,:)	X(6,:)	X(7,:)	X(8,:)	X(9,:)
-2.0	0.0	0.0	2.0	1.0	-5.0	0.0	3.0	0.0
0.0	0.0	0.0	2.0	1.0	-6.0	0.0	3.0	0.0
2.0	0.0	2.0	-2.0	2.0	6.0	0.0	-2.0	0.0
1.0	0.0	0.0	2.0	0.0	6.0	0.0	0.0	0.0
1.0	0.0	2.0	-2.0	1.0	5.0	0.0	-3.0	0.0
0.0	0.0	0.0	-2.0	1.0	-5.0	0.0	3.0	0.0
0.0	-1.0	2.0	-2.0	1.0	-5.0	0.0	3.0	0.0
2.0	0.0	2.0	0.0	1.0	-4.0	0.0	2.0	0.0
1.0	-1.0	0.0	0.0	0.0	4.0	0.0	0.0	0.0
1.0	0.0	0.0	-1.0	0.0	-3.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0	0.0	-4.0	0.0	0.0	0.0
0.0	1.0	0.0	-2.0	0.0	-4.0	0.0	0.0	0.0
1.0	0.0	-2.0	0.0	0.0	4.0	0.0	0.0	0.0
2.0	0.0	0.0	-2.0	1.0	4.0	0.0	-2.0	0.0
0.0	1.0	2.0	-2.0	1.0	3.0	0.0	-2.0	0.0
1.0	1.0	0.0	0.0	0.0	-3.0	0.0	0.0	0.0
1.0	-1.0	0.0	-1.0	0.0	-2.0	0.0	0.0	0.0
-1.0	-1.0	2.0	2.0	2.0	-2.0	0.0	0.0	0.0
0.0	-1.0	2.0	2.0	2.0	-2.0	0.0	0.0	0.0
1.0	-1.0	2.0	0.0	2.0	-3.0	0.0	0.0	0.0
3.0	0.0	2.0	0.0	2.0	-2.0	0.0	0.0	0.0
-2.0	0.0	2.0	0.0	2.0	-3.0	0.0	2.0	0.0
1.0	0.0	2.0	0.0	0.0	3.0	0.0	0.0	0.0
1.0	0.0	0.0	0.0	2.0	-2.0	0.0	0.0	0.0
-1.0	0.0	2.0	-2.0	1.0	-2.0	0.0	0.0	0.0
0.0	-2.0	2.0	-2.0	1.0	-4.0	0.0	2.0	0.0
-2.0	0.0	0.0	0.0	1.0	-2.0	0.0	0.0	0.0
2.0	0.0	0.0	0.0	1.0	2.0	0.0	0.0	0.0
1.0	1.0	2.0	0.0	2.0	2.0	0.0	0.0	0.0

Multiples of Arguments and Coefficients
Wahr Nutation Series for Axis B for Gilbert N = 106
Dziewonski Earth Model 1066A Epoch J2000

X(1,:)	X(2,:)	X(3,:)	X(4,:)	X(5,:)	X(6,:)	X(7,:)	X(8,:)	X(9,:)
0.0	0.0	0.0	0.0	1.0	-171996.0	-174.2	92025.0	8.9
0.0	0.0	2.0	-2.0	2.0	-13187.0	-1.6	5736.0	-3.1
0.0	0.0	2.0	0.0	2.0	-2274.0	-0.2	977.0	-0.5
0.0	0.0	0.0	0.0	2.0	2062.0	0.2	-895.0	0.5
0.0	1.0	0.0	0.0	0.0	1426.0	-3.4	54.0	-0.1
1.0	0.0	0.0	0.0	0.0	712.0	0.1	-7.0	0.0
0.0	1.0	2.0	-2.0	2.0	-517.0	1.2	224.0	-0.6
0.0	0.0	2.0	0.0	1.0	-386.0	-0.4	200.0	0.0
1.0	0.0	2.0	0.0	2.0	-301.0	0.0	129.0	-0.1
0.0	-1.0	2.0	-2.0	2.0	217.0	-0.5	-95.0	0.3
1.0	0.0	0.0	-2.0	0.0	-158.0	0.0	-1.0	0.0
0.0	0.0	2.0	-2.0	1.0	129.0	0.1	-70.0	0.0
-1.0	0.0	2.0	0.0	2.0	123.0	0.0	-53.0	0.0
1.0	0.0	0.0	0.0	1.0	63.0	0.1	-33.0	0.0
0.0	0.0	0.0	2.0	0.0	63.0	0.0	-2.0	0.0
-1.0	0.0	2.0	2.0	2.0	-59.0	0.0	26.0	0.0
-1.0	0.0	0.0	0.0	1.0	-58.0	-0.1	32.0	0.0
1.0	0.0	2.0	0.0	1.0	-51.0	0.0	27.0	0.0
2.0	0.0	0.0	-2.0	0.0	48.0	0.0	1.0	0.0
-2.0	0.0	2.0	0.0	1.0	46.0	0.0	-24.0	0.0
0.0	0.0	2.0	2.0	2.0	-38.0	0.0	16.0	0.0
2.0	0.0	2.0	0.0	2.0	-31.0	0.0	13.0	0.0
2.0	0.0	0.0	0.0	0.0	29.0	0.0	-1.0	0.0
1.0	0.0	2.0	-2.0	2.0	29.0	0.0	-12.0	0.0
0.0	0.0	2.0	0.0	0.0	26.0	0.0	-1.0	0.0
0.0	0.0	2.0	-2.0	0.0	-22.0	0.0	0.0	0.0
-1.0	0.0	2.0	0.0	1.0	21.0	0.0	-10.0	0.0
0.0	2.0	0.0	0.0	0.0	17.0	-0.1	0.0	0.0
0.0	2.0	2.0	-2.0	2.0	-16.0	0.1	7.0	0.0
-1.0	0.0	0.0	2.0	1.0	16.0	0.0	-8.0	0.0
0.0	1.0	0.0	0.0	1.0	-15.0	0.0	9.0	0.0
1.0	0.0	0.0	-2.0	1.0	-13.0	0.0	7.0	0.0
0.0	-1.0	0.0	0.0	1.0	-12.0	0.0	6.0	0.0
2.0	0.0	-2.0	0.0	0.0	11.0	0.0	0.0	0.0
-1.0	0.0	2.0	2.0	1.0	-10.0	0.0	5.0	0.0
1.0	0.0	2.0	2.0	2.0	-8.0	0.0	3.0	0.0
0.0	-1.0	2.0	0.0	2.0	-7.0	0.0	3.0	0.0
0.0	0.0	2.0	2.0	1.0	-7.0	0.0	3.0	0.0
1.0	1.0	0.0	-2.0	0.0	-7.0	0.0	0.0	0.0
0.0	1.0	2.0	0.0	2.0	7.0	0.0	-3.0	0.0

Multiples of Arguments and Coefficients
Wahr Nutation Series for Axis B for Gilbert N = 106
Dziewonski Earth Model 1066A Epoch J2000

X(1,:)	X(2,:)	X(3,:)	X(4,:)	X(5,:)	X(6,:)	X(7,:)	X(8,:)	X(9,:)
-2.0	0.0	0.0	2.0	1.0	-6.0	0.0	3.0	0.0
0.0	0.0	0.0	2.0	1.0	-6.0	0.0	3.0	0.0
2.0	0.0	2.0	-2.0	2.0	6.0	0.0	-3.0	0.0
1.0	0.0	0.0	2.0	0.0	6.0	0.0	0.0	0.0
1.0	0.0	2.0	-2.0	1.0	6.0	0.0	-3.0	0.0
0.0	0.0	0.0	-2.0	1.0	-5.0	0.0	3.0	0.0
0.0	-1.0	2.0	-2.0	1.0	-5.0	0.0	3.0	0.0
2.0	0.0	2.0	0.0	1.0	-5.0	0.0	3.0	0.0
1.0	-1.0	0.0	0.0	0.0	5.0	0.0	0.0	0.0
1.0	0.0	0.0	-1.0	0.0	-4.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0	0.0	-4.0	0.0	0.0	0.0
0.0	1.0	0.0	-2.0	0.0	-4.0	0.0	0.0	0.0
1.0	0.0	-2.0	0.0	0.0	4.0	0.0	0.0	0.0
2.0	0.0	0.0	-2.0	1.0	4.0	0.0	-2.0	0.0
0.0	1.0	2.0	-2.0	1.0	4.0	0.0	-2.0	0.0
1.0	1.0	0.0	0.0	0.0	-3.0	0.0	0.0	0.0
1.0	-1.0	0.0	-1.0	0.0	-3.0	0.0	0.0	0.0
-1.0	-1.0	2.0	2.0	2.0	-3.0	0.0	1.0	0.0
0.0	-1.0	2.0	2.0	2.0	-3.0	0.0	1.0	0.0
1.0	-1.0	2.0	0.0	2.0	-3.0	0.0	1.0	0.0
3.0	0.0	2.0	0.0	2.0	-3.0	0.0	1.0	0.0
-2.0	0.0	2.0	0.0	2.0	-3.0	0.0	1.0	0.0
1.0	0.0	2.0	0.0	0.0	3.0	0.0	0.0	0.0
-1.0	0.0	2.0	4.0	2.0	-2.0	0.0	1.0	0.0
1.0	0.0	0.0	0.0	2.0	-2.0	0.0	1.0	0.0
-1.0	0.0	2.0	-2.0	1.0	-2.0	0.0	1.0	0.0
0.0	-2.0	2.0	-2.0	1.0	-2.0	0.0	1.0	0.0
-2.0	0.0	0.0	0.0	1.0	-2.0	0.0	1.0	0.0
2.0	0.0	0.0	0.0	1.0	2.0	0.0	-1.0	0.0
3.0	0.0	0.0	0.0	0.0	2.0	0.0	0.0	0.0
1.0	1.0	2.0	0.0	2.0	2.0	0.0	-1.0	0.0
0.0	0.0	2.0	1.0	2.0	2.0	0.0	-1.0	0.0
1.0	0.0	0.0	2.0	1.0	-1.0	0.0	0.0	0.0
1.0	0.0	2.0	2.0	1.0	-1.0	0.0	1.0	0.0
1.0	1.0	0.0	-2.0	1.0	-1.0	0.0	0.0	0.0
0.0	1.0	0.0	2.0	0.0	-1.0	0.0	0.0	0.0
0.0	1.0	2.0	-2.0	0.0	-1.0	0.0	0.0	0.0
0.0	1.0	-2.0	2.0	0.0	-1.0	0.0	0.0	0.0
1.0	0.0	-2.0	2.0	0.0	-1.0	0.0	0.0	0.0
1.0	0.0	-2.0	-2.0	0.0	-1.0	0.0	0.0	0.0

Multiples of Arguments and Coefficients
Wahr Nutation Series for Axis B for Gilbert N = 106
Dziewonski Earth Model 1066A Epoch J2000

X(1,:)	X(2,:)	X(3,:)	X(4,:)	X(5,:)	X(6,:)	X(7,:)	X(8,:)	X(9,:)
1.0	0.0	2.0	-2.0	0.0	-1.0	0.0	0.0	0.0
1.0	0.0	0.0	-4.0	0.0	-1.0	0.0	0.0	0.0
2.0	0.0	0.0	-4.0	0.0	-1.0	0.0	0.0	0.0
0.0	0.0	2.0	4.0	2.0	-1.0	0.0	0.0	0.0
0.0	0.0	2.0	-1.0	2.0	-1.0	0.0	0.0	0.0
-2.0	0.0	2.0	4.0	2.0	-1.0	0.0	1.0	0.0
2.0	0.0	2.0	2.0	2.0	-1.0	0.0	0.0	0.0
0.0	-1.0	2.0	0.0	1.0	-1.0	0.0	0.0	0.0
0.0	0.0	-2.0	0.0	1.0	-1.0	0.0	0.0	0.0
0.0	0.0	4.0	-2.0	2.0	1.0	0.0	0.0	0.0
0.0	1.0	0.0	0.0	2.0	1.0	0.0	0.0	0.0
1.0	1.0	2.0	-2.0	2.0	1.0	0.0	-1.0	0.0
3.0	0.0	2.0	-2.0	2.0	1.0	0.0	0.0	0.0
-2.0	0.0	2.0	2.0	2.0	1.0	0.0	-1.0	0.0
-1.0	0.0	0.0	0.0	2.0	1.0	0.0	-1.0	0.0
0.0	0.0	-2.0	2.0	1.0	1.0	0.0	0.0	0.0
0.0	1.0	2.0	0.0	1.0	1.0	0.0	0.0	0.0
-1.0	0.0	4.0	0.0	2.0	1.0	0.0	0.0	0.0
2.0	1.0	0.0	-2.0	0.0	1.0	0.0	0.0	0.0
2.0	0.0	0.0	2.0	0.0	1.0	0.0	0.0	0.0
2.0	0.0	2.0	-2.0	1.0	1.0	0.0	-1.0	0.0
2.0	0.0	-2.0	0.0	1.0	1.0	0.0	0.0	0.0
1.0	-1.0	0.0	-2.0	0.0	1.0	0.0	0.0	0.0
-1.0	0.0	0.0	1.0	1.0	1.0	0.0	0.0	0.0
-1.0	-1.0	0.0	2.0	1.0	1.0	0.0	0.0	0.0
0.0	1.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0

12.11.3 Rotation - B

The B transformation, at *systemtime* $(year, day, sec = t)$, is a counterclockwise rotation about the Celestial Ephemeris Pole, CEP, of time t, through the Greenwich True Sidereal Time, Λ, having been converted to radians. The resulting X axis unit vector will pass through the Greenwich meridian at time t.

$$B = R_3(\Lambda) = \begin{bmatrix} \cos(\Lambda) & \sin(\Lambda) & 0 \\ -\sin(\Lambda) & \cos(\Lambda) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12.81}$$

$$\begin{aligned}
\Lambda &= \text{Greenwich True Sidereal Time (GST) (radians)} \\
\Lambda &= H_0 + \tilde{\omega}(t + \Delta t_2) + \Delta H(t) + (2\pi/S)(.00264\sin(\Omega) + .000063\sin(2\Omega)) \\
T &= \text{Julian centuries from J2000 to 0 hours UT1} \\
GMST &= \text{Greenwich Mean Sidereal Time (0 hours UT1)} \\
GMST &= 24110.54841 + 8640184.812866T + .093104T^2 - .0000062T^3 \quad \text{J2000} \\
GMST &= 24110.4709 + 8640184.7278T + .0929T^2 + 0T^3 \quad \text{B1950} \\
H_0 &= \text{Mean hour angle of Greenwich at 0 hours UT1} \\
H_0 &= AMOD(GMST, 86400)(2\pi/86400) \\
\Delta H(t) &= \text{Equation of the equinoxes } = (2\pi/S)\Delta\psi\cos(\varepsilon) \\
\Delta\psi &= \text{total nutation in longitude (seconds of arc)} \\
\varepsilon &= \text{ true obliquity (degrees) } = \bar{\varepsilon} + \Delta\varepsilon/3600 \\
\bar{\varepsilon} &= \text{ mean obliquity (degrees)} \\
\Delta\varepsilon &= \text{nutation in obliquity (arcseconds)} \\
\tilde{\omega} &= \text{Earth's mean sideral rotation rate in radians/second} \\
&= \text{the product of the ratio of sidereal to universal time and } (2\pi/86400) \\
&= 1.002737909350795(2\pi/86400) \\
&+ (5.9006D(-11)T - 5.9D(-15)T^2)(2\pi/86400) \\
\Delta t_2 &= (UT1 - systemtime) = (UT1 - UTC) - (systemtime - UTC) \\
&= \Delta t + \dot{\Delta t}\cdot t + \ddot{\Delta t}\cdot t^2/2 \\
\dot{\Delta t} &= \ddot{\Delta t} = 0 \text{ initially, and are estimated later.}
\end{aligned}$$

Note that Λ is a function of the Earth orientation parameter $\Delta t_2 = (UT1 - systemtime)$. Thus this parameter will be involved in the Earth gravity calculation and will require partial derivatives, described in section 12.11.5, page 112, to account for dynamic effects if a later process determines Earth orientation.

12.11.4 Polar Motion - A

The polar motion parameters describe the motion of the CEP in the International Terrestrial Reference System (ITRS). The associated transformation converts the CEP to the ITRS reference pole (IRP). This pole is essentially defined by the position of a collection of observing stations whose positions implicitly define the frame for the terrestrial system. The frame is known as the International Terrestrial Reference Frame (ITRF) and the pole is the IRP. Tracking site coordinates would be provided in this frame. See *http://hpiers.obspm.fr/eop-pc/* or the explanations at the sites *http://hpiers.obspm.fr/eop-pc/products/bulletins/explanatory.html* and *http://www.iers.org/iers/earth/*.

This basic polar motion transformation is more easily given by describing the inverse of A, A^T. The transformation A^T carries the IRP into the CEP. The motion is described by providing two angles, Δx and Δy, and carrying out the basic rotation $R_2(\Delta x)$ followed by the basic rotation $R_1(\Delta y)$. The result would be

$$A^T = R_1(\Delta y)R_2(\Delta x) \tag{12.82}$$

where Δx, is measured positively from the IRP along the Greenwich Meridian, i.e., 0° meridian and Δy is measured positively from the IRP along the 90° West meridian.

The required transformation A would then be given by

$$\begin{aligned} A &= [R_1(\Delta y)R_2(\Delta x)]^T = R_2(-\Delta x)R_1(-\Delta y) \\ &= \begin{bmatrix} \cos(\Delta x) & 0 & \sin(\Delta x) \\ 0 & 1 & 0 \\ -\sin(\Delta x) & 0 & \cos(\Delta x) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Delta y) & -\sin(\Delta y) \\ 0 & \sin(\Delta y) & \cos(\Delta y) \end{bmatrix} \end{aligned} \tag{12.83}$$

which, to first order, would be

$$A = \begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & -\Delta y \\ -\Delta x & \Delta y & 1 \end{bmatrix} \tag{12.84}$$

The result would be the same if the calculation were to be carried out in the opposite order, as $R_1(-\Delta y)R_2(-\Delta x)$, and the result only retained to the first order.

The polar motion parameters Δx and Δy, are not used in the calculation of Earth gravity acceleration as it is assumed, within the operational program, that this acceleration should be calculated using the Celestial Ephemeris Pole (CEP). Recall, from the previous section, that the polar motion parameter Δt_2 was involved in the Earth gravity calculation.

In subsequent processing, outside of the OrbGen program, the Earth orientation parameters, their rates, and the constituents of Δt_2 ($\dot{\Delta t}$, $\ddot{\Delta t}$) are estimated. The rates are initialized to zero. Anticipating this process, an evaluation should be made as to *in which frame is the Earth gravity calculation to be carried out*. This will allow for the proper preparation of dynamic Earth orientation partial derivatives in the next section.

If the pole, $\overline{CEP}$, rather than the CEP, is used to evaluate Earth gravity, then satellite perturbations would also be required for Δx and Δy, offsets of the CEP from the IRP in the ITRS. This would allow for any subsequent estimation of the Earth orientation parameters to fully account

for their effect on the orbit. In either case perturbations for the constituents of Δt_2 would be required.

The six year average CEP, i.e., $\overline{CEP}$, is located relative to the IRP by the transformation A_p of equation 11.45, page 44. The $A_p^T A$ transformation takes coordinates in the CEP frame to coordinates in the $\overline{CEP}$ frame. Using the six year average pole would necessitate using this transformation, as going from the CEP to the $\overline{CEP}$ can only be carried out by going through the IRP. It is for this reason that perturbations for Δx and Δy are required, as indicated above.

As discussed in section 11.1, page 39, Earth Gravity, the A_p^T transformation could be implemented through C_{21}, S_{21} in which case A_p^T would not be used and only A would be involved in the actual transformation from inertial to Earth-fixed for the purpose of gravity evaluation. In either implementation Earth orientation parameter perturbations would be required to reflect the gravity computation being carried out in the proper frame.

12.11.5 Earth Orientation Partials

Earth orientation parameter perturbations will be calculated for the parameter set p consisting of Δx, Δy, Δt, $\dot{\Delta t}$, $\ddot{\Delta t}$. Equation 2.5, page 6 is provided here for reference.

$$\partial\ddot{x}/\partial p = (\partial G/\partial x)\partial x/\partial p + (\partial G/\partial\dot{x})\partial\dot{x}/\partial p + (\partial G/\partial p) \qquad (12.85)$$

The only terms that appear in $\partial G/\partial p$ are those which require transformation to the Earth-fixed frame in order to carry out the calculation. This would involve A_{earth} and A_{tide} only. The calculation of $\partial G/\partial p$ is, for $T = ABCD$ rather than BCD, given by

$$\begin{aligned}
G &= T(p)^T G^e = (T(p)/a_e)^T a_e G^e = E a_e G^e \\
\partial G/\partial p &= (\partial T(p)^T/\partial p)G^e + T(p)^T \partial G^e/\partial p \\
\partial G/\partial p &= (\partial T(p)^T/\partial p)G^e + (T(p)/a_e)^T \partial a_e G^e/\partial p \\
\partial T(p)/\partial p &= \partial ABCD/\partial p \\
\partial ABCD/\partial p &= (\partial A/\partial p)BCD \qquad p = (p_1, p_2) = (\Delta x, \Delta y) \\
\partial ABCD/\partial p &= A(\partial B/\partial p)CD \qquad p = (p_3, p_4, p_5) = (\Delta t, \dot{\Delta t}, \ddot{\Delta t}) \\
\partial A/\partial p_i &= \begin{bmatrix} 0 & 0 & \delta_{1i} \\ 0 & 0 & -\delta_{2i} \\ -\delta_{1i} & \delta_{2i} & 0 \end{bmatrix} \qquad i = (1, 2) \\
\partial B/\partial p_i &= \begin{bmatrix} -\sin\Lambda & \cos\Lambda & 0 \\ -\cos\Lambda & -\sin\Lambda & 0 \\ 0 & 0 & 0 \end{bmatrix} \partial\Lambda/\partial p_i \qquad i = (3, 4, 5)
\end{aligned}$$

$$\Lambda = \text{Greenwich True Sidereal Time (GST)}$$

$$\Lambda = H_0 + \Delta H(t) + \tilde{\omega} t + \tilde{\omega}\Delta t_2$$

$$\Delta t_2 = \Delta t + \dot{\Delta t}\cdot t + \ddot{\Delta t}\cdot t^2/2$$

$$
\begin{aligned}
\partial\Lambda/\partial p_i &= \tilde{\omega}(\partial\Delta t_2/\partial p_i) \\
&= \tilde{\omega}(\delta_{3i} + \delta_{4i}t + \delta_{5i}t^2/2) \qquad i = (3,\, 4,\, 5) \\
\partial a_e G^e/\partial p &= \partial a_e G^e_{poletide}/\partial p \qquad p = (p_1,\, p_2) = (\Delta x,\, \Delta y) \\
\partial a_e G^e_{poletide}/\partial p &= \partial\left[\Delta C_{21} a_e \nabla U_2^1 + \Delta S_{21} a_e \nabla V_2^1\right]/\partial p \\
&= \left[\partial\Delta C_{21}/\partial p\right] a_e \nabla U_2^1 + \left[\partial\Delta S_{21}/\partial p\right] a_e \nabla V_2^1 \qquad p = (p_1,\, p_2)
\end{aligned}
$$

using 11.80, page 53 for the Elastic and Anelastic cases.

Taking these calculations into consideration, with the understanding that partials that are not calculated are zero, gives

$$
\begin{aligned}
\partial G/\partial p_i = {}& \frac{(CD)^T}{a_e}\left[B^T \partial A^T/\partial p_i + \partial B^T/\partial p_i A^T\right](a_e G^e) \\
& + \frac{(ABCD)^T}{a_e} \partial a_e G^e_{poletide}/\partial p_i \qquad (12.86) \\
\text{where} \quad & a_e G^e = a_e G^e_{earth} + a_e G^e_{poletide} \qquad (12.87)
\end{aligned}
$$

Note that $A_{poletide} = T^T G^e_{poletide}$ and although this development is for $T = ABCD$, with $E = ABCD/a_e$, the operational OrbGen program uses $T = BCD$, with $E = BCD/a_e$. This will cause some of the above terms to be zero.

Also note that the transformation A, above, could actually be $A_p^T A$, using A_p from equation 11.45, page 44. This would occur if gravity field computation modeling were to use A_p rather than altering C_{21} and S_{21} by equation (11.48).

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14 FILES

14.1 UNIX Script

```
#!/bin/csh -exf
# Run Under C-Shell and Echo Commands
# -x for echo commands, -e to quit at first
#    unsuccessful command
# -f to not search .cshrc file
/bin/date    # Start Time and Date
setenv SWITCH1 1  # Turn on text file switch
setenv SWITCH9 3  # Turn off error reports
# ****************************************
#  BEGIN IBM ORBIT GENERATOR SECTION
# ****************************************
#setenv SWITCH2 0  #Turn on YAW
#setenv SWITCH3 0  #Turn on YAW plus use actual yaw rates
#When producing stk files turn on switch2
#setenv SWITCH5 0  #produce 4 files for STK
#   echo Ephemeris file    > /dev/null
#   setenv FOR77 filename1
#   echo Articulation file > /dev/null
#   setenv FOR78 filename1
#   echo Attitude file     > /dev/null
#   setenv FOR79 filename2
#   echo Time Shape file   > /dev/null
#   setenv FOR80 filename3
echo Process file > /dev/null
setenv FOR2  /omnis/data/inputdata
echo Initial Conditions Scratch File > /dev/null
setenv FOR23 23$$.s
cp /dev/null $FOR23
echo Leap Second File > /dev/null
setenv FOR3 /omnis/data/cleapfl
echo Scratch file > /dev/null
setenv FOR32 32$$.s
echo Earth Orient./ Polar Motion file > /dev/null
#setenv FOR11 /omnis/data/dmapol
setenv FOR11 ./ierspol
echo Temporary Shadow File  > /dev/null
setenv FOR15 15$$.s
cp /dev/null $FOR15
echo Initial Conditions Scratch file  > /dev/null
setenv FOR12 12$$.s
cp /dev/null $FOR12
echo Gravity Field File > /dev/null
```

```
setenv FOR13 /omnis/data/egm9612
echo Sun-moon file > /dev/null
setenv FOR14 /omnis/data/smp9505403
echo Input Geophysical Data file > /dev/null
setenv FOR77 sgdb2000
echo Table of Yaw Biases File > /dev/null
setenv FOR43 /omnis/data/biasfl
echo Yaw Rates file > /dev/null
setenv FOR44 /omnis/data/yawrfl
echo IERS Ocean Tide Model Coeff File > /dev/null
setenv FOR45 /omnis/data/iers96ot.dat
echo IERS Solid-Earth Tabular Data File > /dev/null
setenv FOR46 /omnis/data/iers96se.tab
echo Predicted Mean Pole Model file > /dev/null
setenv FOR48 /omnis/data/mean.pole
echo Satellite Characteristic file > /dev/null
setenv FOR17 /omnis/data/svchar
echo Block2r Radiation Pressure Force File > /dev/null
setenv FOR20 /omnis/data/blk2rfm
echo Output Trajectory file > /dev/null
setenv FOR90 Trajjwo.07
echo Run Orbgen for Sat. 7 > /dev/null
/omnis9/bin/orbgen > Orbgen9out.07
echo Removing Scratch Files > /dev/null
rm -f 12$$.s 15$$.s 23$$.s  32$$.s
# ****************************************
#  END IBM ORBIT GENERATOR SECTION
# ****************************************
#END ORBGEN
echo Purge Process File > /dev/null
/bin/date   # Stop Time and Date
```

14.2 Input Data (unit/IPRFIL/2)

```
*TEXT INPUT FOR ORBGEN
*
VERSION-NGA-R7-0C054-12/04/2000
 .29979245800000D+06 SPEED OF LIGHT
 .00000000000000D+00 SECONDS OF EPOCH
                   7 SATELLITE NUMBER
                2001 YEAR OF EPOCH
                  45 DAY OF EPOCH
                   0 TRAJECTORY TYPE(0-6)(STATE & PARTIALS, ETC)
 .25920000000000D+06 TRAJECTORY SPAN(SECONDS FROM EPOCH)
 .25920000000000D+06 PRINT PREDICTED CONDITIONS (SEC FROM EPOCH)
 .30000000000000D+03 INTEGRATION INTERVAL
 .10000000000000D+02 REDUCED INTEGRATION INTERVAL
 .90000000000000D+03 TRAJECTORY WRITE INTERVAL
 .10000000000000D-05 START ROUTINE POSITION EXIT TOLERANCE
 .10000000000000D-08 START ROUTINE VELOCITY EXIT TOLERANCE
 .10000000000000D-05 STEP REDUCTION TOLERANCE
                   8 NO. OF NEGATIVE TIMELINES
                   0 INTEGRATION TECHNIQUE(0=STD 1=CENTRAL)
                  10 ORDER OF INTEGRATION
                   1 NUMBER OF CORRECTOR INTER. PER TIMELINE
               J2000 PRECESSION NUTATION THEORY
 .10000000000000D+65 YR OF EPOCH FOR INTEGRATION REF FRAME
 .10000000000000D+65 DAY
 .10000000000000D+65 SEC
 .72921158553000D-04 ANGULAR VELOCITY OF THE EARTH
                   0 MEAN=0 OR TRUE=1
                   3 TIME SYSTEM(ET=0, TAI=1, UTC=2, GPS=3)
                   0 READ POLAR MOTION(0=NO,1=IC FILE,2=TRAJ)
                   4 TYPE OF POLAR MOTION(0=DMA,1=IC,2=TRJ,
*                    3=USNO, 4=Tabular Data File)
                2001 YEAR FOR POLAR MOTION COEFFICIENTS
                  31 DAY FOR POLAR MOTION COEFF(DEFAULT=EPOCH)
                   0 IMPROVE POLAR MOTION
                   2 IC SOURCE(0=ICFILE 1=TRAJ 2=COORDINATES)
                   1 0=IMPROVE COORD, 1=IMPROVE ELEMENTS
                  60 ORIGIN OF IC(10=ORBGEN,30=BATCH SOLVER,
*                           40=BATCH PROPAGATOR,50=GPS FILTER,
*                           60=GPS PROPAGATOR,70=OTHER)
                  -1 CYCLE NO FOR IC SEARCH(-1=LAST CYCLE FOR
*                             GIVEN CRITERIA)
                   0 NO OF TERMS FOR ZONAL TIDES(0=NO,41=STD,62)
                   1 APPLY DIURNAL/SEMIDIURNAL CORR(0=NO,1=YES)
-.76947619094252D+04 SATELLITE X COORDINATE
 .24616302036459D+05 SATELLITE Y COORDINATE
```

```
-.54408569845023D+04 SATELLITE Z COORDINATE
-.20207078000128D+01 SATELLITE X COMPONENT OF VELOCITY
-.13433253392457D+01 SATELLITE Y COMPONENT OF VELOCITY
-.30577945956286D+01 SATELLITE Z COMPONENT OF VELOCITY
 .10000000000000D+65 SATELLITE ELEMENT - A
 .10000000000000D+65 SATELLITE ELEMENT - E*SIN(G)
 .10000000000000D+65 SATELLITE ELEMENT - E*COS(G)
 .10000000000000D+65 SATELLITE ELEMENT - I
 .10000000000000D+65 SATELLITE ELEMENT - L+H
 .10000000000000D+65 SATELLITE ELEMENT - H
 .63781370000000D+04 SEMI-MAJOR AXIS OF THE EARTH
 .39860044180000D+06 GM OF EARTH
 .13271240000000D+12 GM OF SUN
 .49027991860000D+04 GM OF MOON
 .10000000000000D+65 SPEED OF LIGHT
                   0 READ GRAVITY FROM IC FILE (0=NO,1=YES)
                   0 EXPANSION AXIS ( 0=CEP, 1=SPIN)
                   0 INCLUDE REL TERMS (0=NO, 1=YES)
                   0 OMIT GRAVITY > 2,2 IN VAR.EQ.(0=NO,1=YES)
                   0 NO. OF GRAVITY ID'S
*                    INSERT GRAVITY ID'S HERE
*
*          DRAG INPUT
 .10000000000000D+65 EARTH SEMI-MAJOR AXIS FOR DRAG MODEL
 .10000000000000D+65 DRAG MODEL OBLATENESS OF EARTH
 .10000000000000D+65 ANGULAR VELOCITY OF THE EARTH
 .10000000000000D+65 SATELLITE MASS (svchar)
 .10000000000000D+65 SAT CROSS SECT AREA FOR DRAG (svchar)
                   0 READ DRAG TIMES AND COEF FROM IC(0=NO,1=YES)
                   0 JACCHIA (0 EXP 1 JACC 2 JACC(350NMCUT) 3 DTM
                   0 JACCHIA OPTION (KPUSE = 0 AP 1 KP)
*
*          DRAG END TIMES (20 VALUES MUST BE ENTERED)
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
```

```
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
*        DRAG COEFFICIENTS (20 VALUES MUST BE ENTERED)
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
  .10000000000000D+65
*
*     RADIATION PRESSURE INPUT
  .69600000000000D+06 RADIUS OF THE SUN
  .17380000000000D+04 RADIUS OF THE MOON
  .14959787069100D+09 ASTRONOMICAL UNIT(KM)
  .45600000000000D-05 SOLAR CONSTANT(NEWTONS/M**2)
  .10000000000000D+65 SAT MASS FOR RADIATION MODEL (svchar)
  .10000000000000D+65 SAT CROSS SECTIONAL AREA (svchar)
  .11107141661465D+01 1ST RAD PRESSURE MODEL PARAMETER
 -.12703746076288D+01 2ND RAD PRESSURE MODEL PARAMETER
  .90000000000000D+02 3RD RAD PRESSURE MODEL PARAMETER
                    0 RADIATION FROM FILE(0=NO,1=YES)
                    2 RADIATION MODEL(0 or NULL=from (svchar)
*                       1=BLK1,2=BLK2A,3=BLK2R,4=SPHERE,
*                       5=NOVA OR 3 PARAMETER,
*                       6=DASENBROCK MODEL
                    3 BLK2A SUBMODEL(0 or -32001=NSWC
*                       1=T20, 2=T20JPL, 3=T20JPL2,
*                       4=TJPLIIA, 5=TJPLIIA2)
                    0 BLK2R SUBMODEL(0 or -32001 = NSWC
```

```
*                       1=T30, 2=T30JPL, 3=TJPLIIR,
*                       4=TJPLIIR2)
*   THRUST INPUT
                    1 BODY FIXED FRAME (0=RVC,1=RAC,2=GPS)
                    0 THRUST VALUES FROM IC/TRAJ(0=NO,1=YES)
* THRUST BEGIN AND END TIMES (4 SET MUST BE ENTERED)
 .10000000000000D+65 BEGIN TIME OF FIRST THRUST
 .10000000000000D+65 END TIME OF FIRST THRUST
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
*
*  THRUST VALUES (X Y Z COMPONETS - 4 SET MUST BE ENTERED)
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
 .10000000000000D+65
* MOMENTUM DUMP VALUES FOR 10 MOMENTUM DUMPS
* (START,STOP,3 ACC COMPONENTS IN GROUPS OF 5)
1.D65                MOMENTUM 1
1.D65                MOMENTUM 2
1.D65                MOMENTUM 3
1.D65                MOMENTUM 4
1.D65                MOMENTUM 5
1.D65                MOMENTUM 6
1.D65                MOMENTUM 7
1.D65                MOMENTUM 8
1.D65                MOMENTUM 9
1.D65                MOMENTUM 10
1.D65                MOMENTUM 11
1.D65                MOMENTUM 12
1.D65                MOMENTUM 13
1.D65                MOMENTUM 14
1.D65                MOMENTUM 15
1.D65                MOMENTUM 16
1.D65                MOMENTUM 17
```

```
 1.D65                    MOMENTUM 18
 1.D65                    MOMENTUM 19
 1.D65                    MOMENTUM 20
 1.D65                    MOMENTUM 21
 1.D65                    MOMENTUM 22
 1.D65                    MOMENTUM 23
 1.D65                    MOMENTUM 24
 1.D65                    MOMENTUM 25
 1.D65                    MOMENTUM 26
 1.D65                    MOMENTUM 27
 1.D65                    MOMENTUM 28
 1.D65                    MOMENTUM 29
 1.D65                    MOMENTUM 30
 1.D65                    MOMENTUM 31
 1.D65                    MOMENTUM 32
 1.D65                    MOMENTUM 33
 1.D65                    MOMENTUM 34
 1.D65                    MOMENTUM 35
 1.D65                    MOMENTUM 36
 1.D65                    MOMENTUM 37
 1.D65                    MOMENTUM 38
 1.D65                    MOMENTUM 39
 1.D65                    MOMENTUM 40
 1.D65                    MOMENTUM 41
 1.D65                    MOMENTUM 42
 1.D65                    MOMENTUM 43
 1.D65                    MOMENTUM 44
 1.D65                    MOMENTUM 45
 1.D65                    MOMENTUM 46
 1.D65                    MOMENTUM 47
 1.D65                    MOMENTUM 48
 1.D65                    MOMENTUM 49
 1.D65                    MOMENTUM 50
*
*  NSWC TIDE MODEL CONSTANTS
 .29000000000000D+00 LOVE'S CONSTANT
              -32001 NMAX (TIDE COEF)
              -32001 MMAX (TIDE COEF)
*
*  IERS TIDE MODEL VALUES
                   1 ELASTIC/ANELASTIC SOLID EARTH
*                       (NONE=0, ANELASTIC=1, ELASTIC=2)
                   8 NMAX FOR OCEAN TIDE COEF.(0-99)
 .50000000000000D-03 H MIN, FOR OCEAN TIDE COEF.(0.0-1.1)
*
*  EARTH TIDE COMPONENTS
                   2 SOLID EARTH TIDE (0=NONE,1=IERS,2=NSWC)
```

```
                    0 PERMANENT TIDE CORRECTION (0=NO,1=YES)
                    0 POLE TIDE CORRECTION (0=NO,1=YES)
                    0 OCEAN TIDE (0=NONE,1=IERS,2=NSWC)
                    0 ATMOSPHERIC TIDE (0=NONE,1=IERS,2=NSWC)
                    3 SUN/MOON (1=SUN,2=MOON,
*                       3=BOTH ACTIVE FOR ATMOS. TIDE)
*
*   FORCES TO USE AND/OR IMPROVE (0=NOT USE,1=USE,
*                                2=USE AND IMPROVE)
                    1 EARTH GRAVITY
                    1 SUN GRAVITY
                    1 MOON GRAVITY
                    0 DRAG
                    2 RADIATION PRESSURE
                    0 THRUST
                    1 TIDES
                    0 PLANETS
* PLANETS TO USE 0=NO, 1=YES
                    0 MERCURY
                    0 VENUS
                    0 MARS
                    0 JUPITER
                    0 SATURN
                    0 URANUS
                    0 NEPTUNE
                    0 PLUTO
*
* GM VALUES FOR PLANENTS
 .22032080500000D+05 GM for MERCURY
 .32485859880000D+06 GM for VENUS
 .42828314300000D+05 GM for MARS
 .12671276785780D+09 GM for JUPITER
 .37940626061100D+08 GM for SATURN
 .57945490071000D+07 GM for URANUS
 .68365340639000D+07 GM for NEPTUNE
 .98160090000000D+03 GM for PLUTO
*
                    2 RESTART OPTION FOR REDUCED STEP INTEGRATION
 .10000000000000D+02 THRUST RESTART INTEGRATION INTERVAL
                    0 ipanel (1 is use solar panels,
*                             0 is do not use solar panels)
 .10000000000000D+65 panel time  1 (packed yrdaysec)
 .10000000000000D+65 panel angle 1 (degrees)
 .10000000000000D+65 panel time  2 (packed yrdaysec)
 .10000000000000D+65 panel angle 2 (degrees)
                .1D65 panel time  3 (packed yrdaysec)
                .1D65 panel angle 3 (degrees)
```

```
             .1D65 panel time  4 (packed yrdaysec)
             .1D65 panel angle 4 (degrees)
             .1D65 panel time  5 (packed yrdaysec)
             .1D65 panel angle 5 (degrees)
             .1D65 panel time  6 (packed yrdaysec)
             .1D65 panel angle 6 (degrees)
             .1D65 panel time  7 (packed yrdaysec)
             .1D65 panel angle 7 (degrees)
             .1D65 panel time  8 (packed yrdaysec)
             .1D65 panel angle 8 (degrees)
             .1D65 panel time  9 (packed yrdaysec)
             .1D65 panel angle 9 (degrees)
             .1D65 panel time 10 (packed yrdaysec)
             .1D65 panel angle10 (degrees)
 .10000000000000D+65 ALTERNATE EARTH GM FOR INITIAL CONDITIONS
*
```

14.3 Leap Second (unit/ILEAP/3)

```
1ST RECORD USED TO TEST IF FILE IS PRESENT
TABLE STARTS WITH YEAR 72. FORMAT(F10.1,2F5.1)
TIME OF LEAP SECOND, MODIFIED JULIAN TIME,YR,DAY
SUBROUTINE TIMSYS ALLOWS FOR 40 TIMES AT PRESENT
40952.0  71     1.
41317.0  72.    1. (leap)
41499.0  72. 183. (leap)
41683.0  73.    1.
42048.0  74.    1.
42413.0  75.    1.
42778.0  76.    1. (leap)
43144.0  77.    1.
43509.0  78.    1.
43874.0  79.    1.
44239.0  80.    1. (leap)
44786.0  81. 182.
45151.0  82. 182.
45516.0  83. 182.
46247.0  85. 182.
47161.0  88.    1. (leap)
47892.0  90.    1.
48257.0  91.    1.
48804.0  92. 183. (leap)  (july 1,1992,0 sec of day)
49169.0  93. 182.         (july 1,1993,0 sec of day)
49534.0  94. 182.         (july 1,1994,0 sec of day)
50083.0  96.    1. (leap) (jan  1,1996,0 sec of day)
50630.0  97. 182.         (july 1,1997,0 seconds of day)
51179.0  99.    1.        (jan  1,1998,0 seconds of day)
53187.0  04. 183. (leap)  (limiting record, not yet a leap sec)
00000.0  00.    0.        end of read loop marker
```

14.4 Geopotential Coefficients (unit/IGVFIL/13)

```
RECORD #               DESCRIPTION
1                      IDENTIFYING INFORMATION, SUCH AS WGS84
                       FORMAT (A40)
 RECORD #              DESCRIPTION
2 - N                  FORMAT(5X, 2I5, 2D20.14)
N = number of last record

COLUMN #               DESCRIPTION
6-10                   M(I)     -  ORDER OF C AND S TERMS
11-15                  N(I)     -  DEGREE OF C AND S TERMS
16-35                  C(I)- C  -  COSINE FOR GRAVITY EXPANSION
36-55                  S(I)- S  -  SINE FOR GRAVITY EXPANSION
Partial File Example
CS                        89.                12.                    12.
260      0     0 .10000000000000E+01 .00000000000000E+00
260      0     2-.10826266835532E-02 .00000000000000E+00
260      1     2-.24144104668855E-09 .15431232114442E-08
260      2     2 .15744603745640E-05-.90380380663856E-06
260      0     3 .25326564853322E-05 .00000000000000E+00
260      1     3 .21926385291686E-05 .26842489029678E-06
260      2     3 .30898920688051E-06-.21143761243734E-06
260      3     3 .10054877806438E-06 .19722255900591E-06
260      0     4 .16196215913670E-05 .00000000000000E+00
260      1     4-.50879936040383E-06-.44914487283934E-06
260      2     4 .78417585984376E-07 .14817786829561E-06
260      3     4 .59209940262913E-07-.12007766763363E-07
260      4     4-.39840741176627E-08 .65257142537043E-08
260      0     5 .22729608286870E-06 .00000000000000E+00
260      1     5-.53180301500760E-07-.80858694766073E-07
260      2     5 .10558716839090E-06-.52329193621634E-07
260      3     5-.14930063749229E-07-.70973423688954E-08
260      4     5-.22993002901270E-08 .38671233585094E-09
260      5     5 .43082246205179E-09-.16481826262807E-08
260      0     6-.54068123910708E-06 .00000000000000E+00
260      1     6-.59865669865304E-07 .20684117508018E-07
260      2     6 .59929123854365E-08-.46493035708214E-07
260      3     6 .11854207265132E-08 .18716368503814E-09
260      4     6-.32636177805233E-09-.17845028500909E-08
260      5     6-.21559410691195E-09-.43298133715665E-09
260      6     6 .22543497083101E-11-.55260936645408E-10
260      0     7 .35235990841824E-06 .00000000000000E+00
```

14.5 Initial Conditions (unit/IICFIL/12)

RECORD TYPE	DESCRIPTION
1	FILE HEADER (40 CHAR ALPHA FILELDS)

FIELD #	DESCRIPTION
1	FILE NAME
2	OTHER IDENTIFYING INFORMATION

NOTE: The following records are repeated for each set of initial conditions written to the file.

RECORD TYPE	DESCRIPTION
2	HEADER FOR INITIAL CONDITIONS SET

WORD	DESCRIPTION
1	RECORD TYPE = 2
2	NO. OF INTEGERS IN RECORD
3	NO. OF REALS IN RECORD
4	NO. OF SATELLITES IN THIS SET (NOSAT)
5	SATELLITE NO. OF 1ST SET OF COND.
.	
.	
.	
(NOSAT+4)	SATELLITE NO. OF LAST SET OF COND.
(NOSAT+9)	NO. OF POLAR MOTION RECORDS
(NOSAT+10)	NO. OF IMPROVED GRAVITY PARAMETERS
(NOSAT+11)	NO. OF OTHER IMPROVED PARAMETERS (NOTHR)
(NOSAT+12)	ID OF ORIGIN OF INITIAL CONDITIONS (E.G., ORBGEN, SOLUTION, FILTER, PROP)
(NOSAT+13)	IMPROVEMENT CYCLE NUMBER
(NOSAT+14)	DATE OF RUN
(NOSAT+15)	TIME CLOCK VALUE OF RUN)
(NOSAT+16)	EPOCH - YEAR
(NOSAT+17)	EPOCH - DAY
(NOSAT+18)	EPOCH - SEC

NOTE: This record appears for each satellite in a run and is identified above in Record Type 2.

RECORD TYPE	DESCRIPTION
3	INITIAL CONDITIONS RECORD

WORD	DESCRIPTION
1	RECORD TYPE = 3
2	NO. OF INTEGERS IN RECORD
3	NO. OF REALS IN RECORD

```
4                       SATELLITE NO.
5                       NO. OF DRAG PARAMETERS (NDPA)
6                       NO. OF THRUST (NTPA)
7                       RADIATION PRESSURE COMPONENTS (NRPA)
8                       EPOCH   - YEAR
9                       EPOCH - DAY
10                      EPOCH - SEC
11 - )
.    )
.    )                  INITIAL CONDITIONS (WHOLE ARRAY)
.    )                  (POS., VEL., DRAG., THRUST, RAD. PRES.)
(NOIC +11) - )
(L* + 1)                1ST DRAG TIME
.
.
.
(L* + NDPA)             LAST DRAG TIME
(M** + 1)               START TIME OF 1ST THRUST
(M** + 2)               END TIME OF 1ST THRUST
.
.
.
(M** + NTPA)   START TIME OF LAST THRUST TIME
(M** + NTPA)   END TIME OF LAST THRUST TIME

Where,
       *   L = NOIC + 7
       ** M = NOIC + 7 + NDPA
       NOIC = Dimension of the Initial Conditions Array
        (= 41 at present time)

RECORD TYPE             DESCRIPTION
4                       PLOAR MOTION RECORD

NOTE: This type record appears for each group of polar motion
values used in the run.  A group of values consists of eleven
days of values.

WORD                    DESCRIPTION
1                       RECORD TYPE = 4
2                       NO. OF INTEGER VALUES IN RECORD
3                       NO. OF REAL VALUES IN RECORD
4                       PACKED WORD DZZGGGGGPP, WHERE
                        D = DIURNAL EARTH ORIENTATION EFFECTS
                        FLAG, 0=NO,1=YES
                        ZZ = NUMBER OF ZONAL TIDE TERMS
                        GGGGG = GENERATION DATE (MJD)
```

	USED TO GENERATE POLAR MOTION VALUES
	PP = SOURCE OF POLAR MOTION VALUES
	10 = COEF. FILE
	11 = IMPROVED VALUES FROM COEF. FILE
	20 = USNO COEF.
	21 = IMPROVED USNO COEF., ETC.
5	YEAR OF POLAR MOTION VALUE
6	DAY OF POLAR MOTION VALUE
7	POLAR MOTION - DELTA P (RADIANS)
8	POLAR MOTION - DELTA Q (RADIANS)
9	POLAR MOTION - DELTA T (UT1-UTC)(SEC.)
.	
.	
.	
(6-9)	REPEAT FOR 10 MORE DAYS

NOTE: This record appears if gravity parameters were improved. Record type 2 contains the number of gravity parameters, and if this number is 0, this record will not appear.

RECORD TYPE	DESCRIPTION
5	GRAVITY PARAMETER RECORD
WORD	DESCRIPTION
1	RECORD TYPE = 5
2	NO. OF INTEGERS IN RECORD
3	NO. OF REALS IN RECORD
(4-NGRV+3)	GRAVITY ID (I), (I = 1, NGRV)
(NGRV + 4 - ...)	IMPROVED GRAVITY COEFFICIENT (I) (I = 1, NGRV)

NOTE: Record appears if NOTHR in Record Type 2 not equal zero.

RECORD TYPE	DESCRIPTION
6	OTHER IMPROVED PARAMETER RECORDS

WORD	DESCRIPTION
1	RECORD TYPE = 6
2	NO. OF INTEGERS IN RECORD
3	NO. OF REALS IN RECORD
(4-NOTHR+3)	PARAMETER LABEL (I), (I = 1, NOTHR)
(NOTHR - ...)	IMPROVED OTHER PARAMETER (I), (I = 1, NOTHR)

14.6 NGA Earth Orientation Coefficients (unit/IPMFIL/11)

RECORD #	DESCRIPTION
1	FORMAT (F10.2, 6F10.6, F6.2, 4X)

ITEM POSITION	FORMAT	ITEM NAME
1	F10.2	TA
11	F10.6	A
21	F10.6	B
31	F10.6	C1
41	F10.6	C2
51	F10.6	D1
61	F10.6	D2
71	F6.2	P1
77	4X	FILL

RECORD #	DESCRIPTION
2	FORMAT (F6.2,6F10.6,2F6.2,2X)

ITEM POSITION	FORMAT	ITEM NAME
1	F6.	P2
7	F10.6	E
17	F10.6	F
27	F10.6	G1
37	F10.6	G2
47	F10.6	H1
57	F10.6	H2
67	F6.2	Q1
77	F6.2	Q2
79	2X	FILL

RECORD #	DESCRIPTION
3	FORMAT (F10.2, 6F10.6, 10X)

ITEM POSITION	FORMAT	ITEM NAME
1	F10.2	TB
11	F10.6	I
21	F10.6	J
31	F10.6	K1
41	F10.6	K2
51	F10.6	K3
61	F10.6	K4
71	10X	FILL

RECORD #	DESCRIPTION
4	FORMAT (4F10.6, 4F9.4, 4X)

ITEM POSITION	FORMAT	ITEM NAME
1	F10.6	L1
11	F10.6	L2
21	F10.6	L3
31	F10.6	L4
41	F9.4	R1
50	F9.4	R2
59	F9.4	R3
68	F9.4	R4
77	4X	FILL

RECORD #	DESCRIPTION
5	FORMAT (I4, I5, I6, 1X, I6, 58X)

ITEM POSITION	FORMAT	ITEM NAME
1	I4	DELTA AT
5	I5	BULLETIN SERIAL NO.
10	I6	EFFECTIVITY DATE
16	1X	FILL
17	I6	GENERATION DATE
35	58X	FILL

14.7 Earth Orientation Values (unit/IPMFIL/11)

```
One record per day, (2F5.0, 3D20.14), for some span of days.
WORD            DESCRIPTION
1               YEAR
2               DAY NO.
3               DELTA P
4               DELTA Q
5               DELTA T (THIS DELTA T, NOT -OMEGA DELTA T)
```

14.8 Solar Geophysical Database (unit/INGEO/77)

Following is a sample of the GEO database for Canadian and Air Force data:

```
CANA
49291 1993 10 31  304  91.5    94   0   0   3   2   1   1   1   2

AFSF
49278 1993 10 18  291  87.0    95   4  15   7   7   7   7   4   4
49279 1993 10 19  292  86.0    95   4   4   4   4   4   4   4   4
49280 1993 10 20  293  85.0    95   2   2   2   2   2   2   2   2
49281 1993 10 21  294  84.0    95   0   0   0   0   0   0   0   0
```

The format follows:

LINE #	DESCRIPTION
1	CANA OR AFSF

LINE #	DESCRIPTION
2 - N	SEE BELOW FOR DESCRIPTION

FIELD	FORMAT	DESCRIPTION
1	I5	MODIFIED JULIAN DATA
2	I5	YEAR
3	I3	MONTH
4	I3	DAY OF MONTH
5	I5	DAY OF YEAR
6	F6.1	FLUX
7	I6	AVERAGE VALUE
8-15	8I4	KP VALUES IF CANADIAN DATA OR AP VALUES IF AIR FORCE DATA

14.9 Sun-Moon (unit/ISMFIL/14)

RECORD TYPE
1

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	RTYPE	RECORD AND FILE ID (110XXX), WHERE 110 IS RECORD ID AND XXX IS FILE NO. IN ASCENDING ORDER BY YEAR
2	F	YEAR	YEAR OF SUN AND MOON DATA
3	F	RDD	NO. OF DAY RECORDS IN YEAR

RECORD TYPE
2

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	RTYPE	RECORD AND FILE ID (111XXX)
2	F	DAY	DAY NUMBER OF DATA
3-5	F	XS(I)I=1,3	COORDS OF SUN AT ONE DAY INTERVALS
6-8	F	XM(I)I=1,3	COORDS OF MOON AT HALF DAY INTERVAL
9-11	F	XM(I)I=1,3	COORDS OF MOON AT HALF DAY INTERVAL

NOTE: Record Type 2 is repeated RRD times.

NOTE: Last record of Type 2 is a Dummy record with negative value in Word 1.

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	RTYPE	ID (-119) IND. END OF YEAR
2-20	F	DUM	DUMMY WORDS

NOTE: Record Types 1 and 2 are repeated for additional years of data.
END OF INFORMATION RECORD

WORD NO.	TYPE	SYMBOLIC NAME	DESCRIPTION
1	F	RTYPE	ID (-199999) IND. END OF ALL DATA
2	F	FYEAR	FINAL YEAR ON FILE
3	F	TYEAR	NO. OF YEARS ON FILE

14.10 Trajectory (unit/ITJFIL/90)

RECORD TYPE	DESCRIPTION
1	FILE HEADER, 40 CHARACTER ALPHA FIELDS

FIELD	TYPE	DESCRIPTION
1	C	FILE NAME
2	C	GRAVITY MODEL FROM COEFFICIENT FILE
3	C	TIDE MODEL FROM COEFFICIENT FILE
4	C	DENSITY MODEL
5	C	RADIATION MODEL
6	C	POLE ID
7	C	BODY-FIXED AXIS SYSTEM FOR ANTENNA
8	C	BODY-FIXED AXIS SYSTEM FOR THRUST
9	C	TRAJECTORY FORMAT VERSION (08/31/93)
10	C	NUMBER OF MERGED TRAJECTORIES IN CHARACTER FORMAT (I5,35H)

RECORD TYPE	DESCRIPTION
2	HEADER, TRAJECTORY INFORMATION

WORD	TYPE	DESCRIPTION
1	I	HEADER RECORD ID = 2
2	I	NUMBER OF INTEGERS
3	I	NUMBER OF REALS
4	I	TYPE OF TRAJ+NUMBER OF EXTRA TIMELINES 108=INTEGRATED TRAJECTORY (8 EXTRA LINES) TIME LINES BEFORE EPOCH AND BEYOND TRAJ. SPAN USED FOR INTERPOLATION 204=BATCH PROP TRAJECTORY(4 EXTRA LINES) 304=SEQ/GPS PROP TRAJ(4 EXTRA LINES) 408=NAV MESSAGE TRAJ(8 EXTRA LINES)
5	I	SATELLITE NUMBER
6	I	MEAN/TRUE(0/1)
7	I	EXPANSION AXIS(0,1,2,..., 0 IMPLIES NO POLE PARTIALS, OTHERWISE 4 POLE PARTIALS)
8	I	TIME SYSTEM FLAG(0,1,2,...)
9	I	NUMBER OF POLE RECORDS(SUB-HEADER 2)
10	I	NUMBER OF DRAG PARAMETERS (NDPA)
11	I	NUMBER OF THRUST PARAMETERS (NTPA)
12	I	NUMBER OF RADIATION PARAMETERS (NRPA)
13	I	NUMBER OF GRAVITY PARAMETERS (NGPA)
14	I	TOTAL NUMBER OF OTHER PARAMETERS (NOTHR) (LOOK IN ISPARE ARRAY AT WORD NPAR+26 FOR TYPE OF PARAMETER AND HOW MANY OF EACH)
15	I	DATE OF INTEGRATION(YYMMDD)

16	I	TIME OF INTEGRATION(HHMMSS)
17	I	TRAJECTORY REFERENCE POINT(0=CM,1,2,...)
18	I	NUMBER OF EARTH-FIXED ITEMS IN TABULAR RECORD
19	I	NUMBER OF INERTIAL ITEMS IN TABULAR RECORD
20	I	NUMBER OF PERTURBED PARAMETERS(NPAR) IN TABULAR RECORD
21	I	NUMBER OF OTHER ITEMS IN TABULAR RECORD(NOTHER) (INCLUDES 8X8 COVARIANCE MATRIX IF MERGED TRAJ)
22	I	PARAMETER LABEL(I) (I=1,NPAR)
.		
.		
NPAR +22	I	PARTIALS WRT ELEMENTS=1, COORDINATES=0
NPAR +23	I	ORDER OF INTEGRATION
NPAR +24	I	IMPROVEMENT CYCLE NUMBER
NPAR +25	I	NUMBER OF SPARE(OR FUTURE) INTEGER VALUES(NSPR)
NPAR +26	I	ARRAY OF SPARE INTEGER VALUES(I) (I=1,NSPR)
.		ISPARE(1) - SIZE OF IC ARRAY,NOI
.		ISPARE(2) - EARTH GRAVITY USED (0=NO,1=YES)
.		ISPARE(3) - SUN GRAVITY USED (0=NO,1=YES)
.		ISPARE(4) - MOON GRAVITY USED (0=NO,1=YES)
.		ISPARE(5) - DRAG USED (0=NO, 1=YES)
.		ISPARE(6) - RADIATION PRESSURE USED (0 OR 1)
.		ISPARE(7) - THRUST USED (0=NO,1=YES)
.		ISPARE(8) - TIDES USED (0=NO,1=YES)
.		ISPARE(9) - NUMBER MOMENTUM DUMPS (VALUES AND TIME IN RECORD TYPE 6)
.		ISPARE(10)- DENSITY MODEL (0=STD,1=JACCHIA)
.		
(N = NPAR + NSPR)		
N +26	R	INERTIAL-EARTH-FIXED TRANSFORMATION THEORY
(		J2000 OR B1950)(CONTAINS 2000. OR 1950.)
N +27	R	EPOCH YEAR
N +28	R	EPOCH DAY
N +29	R	EPOCH SECOND
N +30	R	MODIFIED JULIAN DATE
N +31	R	DRAG TIME FOR EACH DRAG (SECS FROM EPOCH)
.		(DTIM(I),I=1,NDPA)
.		
		(L = NPAR + NSPR + NDPA)
L +31	R	START TIME OF 1ST THRUST (SECS FROM EPOCH)
L +32	R	END TIME OF 1ST THRUST (SECS FROM EPOCH)
.		(TTIMS(I,1),TTIMS(I,2),I=1,NTPA)
.		

(M = L + 2*NTPA)

M +31	R	WRITE INTERVAL (SECONDS)
M +32	R	FIRST INTEGRATION INTERVAL (SECONDS)
M +33	R	SECOND INTEGRATION INTERVAL (SECONDS)
M +34	R	TRAJECTORY SPAN (SECONDS FROM EPOCH)
M +35	R	EARTH GEOPOTENTIAL MODEL ELLIPSOID RADIUS
M +36	R	EARTH DENSITY MODEL ELLIPSOID RADIUS
M +37	R	EARTH DENSITY MODEL ELLIPSOID OBLATENESS
M +38	R	EARTH ANGULAR VELOCITY (EF TO INERTIAL TRANS)
M +39	R	EARTH ANGULAR VELOCITY FOR DENSITY MODEL
M +40	R	MASS OF SATELLITE FOR DENSITY MODEL
M +41	R	MASS OF SATELLITE FOR RADIATION MODEL
M +42	R	CROSS SEC AREA OF SAT(DENSITY MODEL)
M +43	R	CROSS SEC AREA OF SAT(RADIATION)
M +44	R	LOVE'S CONSTANT FOR TIDE MODEL
M +45	R	EARTH MU
M +46	R	MOON MU
M +47	R	SUN MU
M +48	R	RADIUS OF THE SUN
M +49	R	RADIUS OF THE MOON
M +50	R	SPEED OF LIGHT
M +51	R	INITIAL CONDITIONS SET (WHOLE ARRAY) (POSITION & VELOCITY, DRAGS,THRUST,RAD. PRES.)
.		
.		
.		
M +50+NOI	R	LAST VALUE IN INITIAL CONDITIONS ARRAY (WHERE NOI=SIZE OF INITIAL COND. ARRAY)
M +50+NOI+1	R	INERTIAL FRAME DATE YEAR(IF NULL, THEORY DATE}
M +50+NOI+2	R	INERTIAL FRAME DATE DAY (2000 OR 1950 USED)
M +50+NOI+3	R	INERTIAL FRAME DATE SEC
M +50+NOI+4	R	EPOCH OF 1ST MERGED TRAJ (MOD JULIAN (Y,D,S))
M +50+NOI+5	R	EPOCH OF 2ND MERGED TRAJ (MOD JULIAN (Y,D,S))
.		
.		
M +50+NOI+6	R	EPOCH OF NTH MERGED TRAJ (MOD JULIAN (Y,D,S))

RECORD TYPE	DESCRIPTION
3	SUB- HEADER 2, POLAR MOTION VALUES

NOTE: THIS RECORD IS REPEATED FOR THE NUMBER OF POLE RECORDS INDICATED IN RECORD 2, WORD 9.

WORD	TYPE	DESCRIPTION

1	I	POLE RECORD ID = 3
2	I	NUMBER OF INTEGERS
3	I	NUMBER OF REALS
4	I	PACKED WORD CONTAINING (DZZJJJJJII)
		D = DIURNAL EFFECTS FLAG (0,1)
		ZZ = NUMBER OF ZONAL TIDE COEF.
		JJJJJ = GENERATION DATE OF COEFFICIENT USED (MOD.JULIAN)
		II = SOURCE OF COEF. OR VALUES
		= 10 - NGA COEFFICIENTS
		= 11 - IMPROVED NGA VALUES
		= 20 - USNO COEFFICIENTS
		= 21 - IMPROVED USNO VALUES
		= 40 - BIH FINAL VALUES
		ETC.
5	R	YEAR OF POLAR MOTION VALUES

NOTE: THE NEXT 4 WORDS ARE REPEATED ELEVEN TIMES WITHIN THIS RECORD

6	R	DAY OF POLAR MOTION VALUE
7	R	DELTA P (RADIANS)
8	R	DELTA Q (RADIANS)
9	R	DELTA T (UT1-UTC) IN SEC.

RECORD TYPE	DESCRIPTION
4	SUB-HEADER 3, ANTENNA INFORMATION

WORD	TYPE	DESCRIPTION
1	I	ANTENNA RECORD ID = 4
2	I	NUMBER OF INTEGERS = 0
3	I	NUMBER OF REALS = 15
4	R	X COMPONENT FOR CENTER OF MASS CHANGE (KM)
5	R	Y COMPONENT FOR CENTER OF MASS CHANGE (KM)
6	R	Z COMPONENT FOR CENTER OF MASS CHANGE (KM)
7	R	X COMPONENT FOR ANTENNA 1 OFFSET (KM)
8	R	Y COMPONENT FOR ANTENNA 1 OFFSET (KM)
9	R	Z COMPONENT FOR ANTENNA 1 OFFSET (KM)
.		.
.		.
.		.
18	R	Z COMPONENT FOR ANTENNA 4 OFFSET (KM)

RECORD TYPE	DESCRIPTION
5	SUB-HEADER 4, GRAVITY PARAMETER INFORMATION

This record appears if the gravity parameters are improved.

Record type 2, Word 13 contains the number of gravity parameters, and if this number is 0, this record does not appear.

WORD	DESCRIPTION
1	RECORD ID = 5
2	NUMBER OF INTEGERS IN RECORD
3	NUMBER OF REALS IN RECORD
(4 - NGPA+3)	GRAVITY ID(I), (I=1,NGPA)
(NGPA+4 - ...)	GRAVITY COEFFICIENT (I),(I=1,NGPA)

RECORD TYPE	DESCRIPTION
6	SUB-HEADER 5, OTHER PARAMETERS TO IMPROVE

NOTE: This record appears if NOTHR (word 14) in Record Type 2 not equal zero.

WORD	TYPE	DESCRIPTION
1	I	RECORD ID = 6
2	I	NUMBER OF INTEGERS IN RECORD
3	I	NUMBER OF REALS IN RECORD
4	I	PARAMETER LABEL(I),(I=1,NOTHR)
.		
NOTHR+4	R	START TIME OF 1ST MOMENTUM DUMP
NOTHR+5	R	END TIME OF 1ST MOMENTUM DUMP
		(DMOMTM(I,1),DMOMTM(I,2),I=ISPARE(9))
.		
.		
(M = NOTHR+4 + 2*ISPARE(9))		
M	R	1ST COMPONENT OF 1ST MOMENTUM DUMP
M + 1	R	2ND COMPONENT OF 1ST MOMENTUM DUMP
M + 2	R	3RD COMPONENT OF 1ST MOMENTUM DUMP
		(REPEATED FOR EACH MOMENTUM DUMP), I.E.
		(DMOMAC(I),I=1, 3*ISPARE(9))

NOTE: Record type 7 is repeated for each time line of the trajectory.

RECORD TYPE	DESCRIPTION
7	TABULAR DATA

FIELD	TYPE	DESCRIPTION
1	I	TIME LINE RECORD ID = 10
2	I	NUMBER OF INTEGERS
3	I	NUMBER OF REALS
4	R	TIME FROM EPOCH

NOTE: Any of the following items are optional, depending upon the number of the item specified in Record Type 2. If the number of the item equals 0, then the item does not appear.

5	R	EARTH-FIXED ITEMS (NONE, POS. & VEL. VECTORS, OR IF PROPAGATED TRAJECTORY CM POS & VEL + ANTENNA POS & VEL : TO INTERPOLATE FOR CM & ANTENNA USE ICASE = 22 IN CALL TO NTERP8) (IF IRIS TRAJ, POS & VEL, BCD MATRIX, P, AND Q)
6	R	INERTIAL ITEMS (NONE OR POSITION VECTOR)
7	R	PARTIALS (NONE OR 3*NPAR; POS. PARTIALS ONLY))
8	R	OTHER ITEMS,E.G.,13 GPS ITEMS,(3 BODY-FIXED RAD. PRESS. ACCEL, 3X3 TRAN. MATRIX FOR BODY FIXED TO INERTIAL, AND SHAPE FACTOR) (IF HV, THEN IN ADDITION TO ABOVE THE, 3 DRAG ACCELERATIONS, 3 SATELLITE SUB POINT VALUES, AND THE SUN POSITION VECTOR) (IF MERGED TRAJ, 13 GPS ITEMS + 8X8 COVARIANCE MATRIX)

RECORD TYPE	DESCRIPTION
8	FLAG RECORD

NOTE: This is a flag record and is a repeat of the last tabular record with the time set to a large value (1.D64).

NOTE: This type record is written each time the satellite transitions into or out of shadow. The information is written to a temporary file during the integration process. (See Shadow File format)
Then at the end of the integration process, the trajectory is positioned to the end of information, and the above flag record is written, then the temporary shadow file is read and the shadow information is copied to the trajectory file.
This is all done by subroutine PUTSHD. Programs that need the shadow information get it from the trajectory by calling a routine, GETSHD, which reads the trajectory and writes the shadow information to a temporary file.

RECORD TYPE	DESCRIPTION
9	SHADOW DATA

FIELD	TYPE	DESCRIPTION
1	A	RADIATION PRESSURE MODEL NAME
2	I	SATELLITE NUMBER
3	A	SHADOW TYPE (CLEAR>PENUM, PENUM>UMBRA, UMBRA>PENUM, PENUM>CLEAR)

4	R	TIME, SEC FROM EPOCH
5	R	PERCENT OF EARTH SHADOW
6	R	PERCENT OF MOON SHADOW
7	R	YEAR (4-DIGIT YEAR I.E. 1999)
8	R	DAY OF YEAR
9	R	SECONDS OF DAY
10	R	MODIFIED JULIAN DATE

14.11 Shadow (unit/ISHDFL/15)

The Shadow file is a text file containing times the satellite enters and exits shadow and/or partial shadow. The information is written to a temporary file in the format described below. This information is then appended to the trajectory by the subroutine PUTSHD. Programs that need the shadow information can get it by calling the subroutine GETSHD, which reads the information from the trajectory file and writes it to a temporary file as described below. The Files List program will also extract this information from the trajectory file.

FORMAT	VARIABLE	DESCRIPTION
A6	CID	RAD PRESS MODEL(SEE BELOW)
1X		
I5	ISAT	SATELLITE NUMBER
1X		
A11	CIND	SHADOW INDICATOR(SEE BELOW)
1X		
F13.6	TIME	SEC FROM EPOCH
1X		
F5.1	PEARTH	PERCENT OF EARTH SHADOW
1X		
F5.1	PMOON	PERCENT OF MOON SHADOW
1X		
F5.0	YR	YEAR
1X		
F4.0	DAYR	DAY OF YEAR
1X		
F12.6	SEC	SECOND OF THE DAY
1X		
F6.0	XMJD	MODIFIED JULIAN DATE

NOTE: CID IS THE RADIATION PRESSURE MODEL:

BLOCK1
BLOCK2
BLK2R
SPHERE
NOVA
DASEN
T20
T30
T20JPL
T20JPL2
NONE

```
NOTE:  CIND IS THE SHADOW INDICATOR:
       CLEAR>PENUM
       PENUM>UMBRA
       UMBRA>PENUM
       PENUM>CLEAR
```

Example:

```
BLOCK1  5 CLEAR>PENUM  112084.191964 100.0 100.0 1997.
231. 86399.123456 12345.
BLOCK1  5 PENUM>UMBRA  112199.406964 100.0    .0 1997.
231. 86399.123456 12345.
BLOCK1  5 UMBRA>PENUM  113976.225901 100.0    .0 1997.
231. 86399.123456 12345.
```

14.12 Satellite Characteristics (unit/ISATFL/17)

The file consists of groups of seven (7) records for each satellite. The first record is a header, the second is for the Data Prep and OrbGen modules, the third is the satellite's center of mass offsets, and the last four contain antenna offsets for up to four different antennas. The antennas are identified by the order of the records. Enter all records even if the satellite does not have four antennas.

RECORD #
1

COLUMN	FORMAT	TYPE	DESCRIPTION
1-4	I4	I	SATELLITE ID
6-9	A4	A	FRAME ID (XYZ - BODY-FIXED SYSTEM) (??? - TO BE DEFINED)
11-18	A8	A	RADIATION PRESSURE FORCE MODEL (BLOCK1, BLOCK2, SPHERE, NOVA)
19-30	D12.6	R	SATELLITE MASS FOR RADIATION PRESSURE MODEL (KG)
31-42	D12.6	R	SATELLITE CROSS SECTIONAL AREA FOR RADIATION PRESSURE MODEL (KM**2)
43-54	D12.6	R	SATELLITE MASS FOR DRAG FORCE MODEL (KG)
55-66	D12.6	R	SATELLITE CROSS SECTIONAL AREA FOR DRAG MODEL (KM**2)

RECORD #
2

COLUMN	FORMAT	TYPE	DESCRIPTION
1-5	I5	I	APL SATELLITE ID
6-10	I5	I	SPASUR SATELLITE ID
11-20	D10.4	R	SATELLITE OFFSET FREQUENCY
21-32	D12.6	R	SOLAR PANEL SIZE (KM**2)
33-44	D12.6	R	SOLAR AZIMUTH ROTATION LIMIT

NOTE: THIS LIMIT IS LESS THAN HALF PI AND 0 MEANS NO LIMIT ON SOLAR PANEL ROTATION

45-56	D12.6	R	LENGTH OF SATELLITE BODY (USED WITH DASENBROCK RADIATION MODEL)(KM)
57-68	D12.6	R	WIDTH OF SATELLITE BODY (FOR DASENBROCK) (KM)
69-80	D12.6	R	HEIGHT OF SATELLITE BODY (FOR DASENBROCK) (KM)

RECORD #
3

COLUMN	FORMAT	TYPE	DESCRIPTION
1-5	I5	I	ANTENNA ID = 0
6-25	D20.14	R	X OF CM CHANGE (KM)
26-45	D20.14	R	Y OF CM CHANGE (KM)
46-65	D20.14	R	Z OF CM CHANGE (KM)

RECORD #
4

COLUMN	FORMAT	TYPE	DESCRIPTION
1-5	I5	I	ANTENNA ID = 1
6-25	D20.14	R	X OF ANTENNA OFFSET (KM)
26-45	D20.14	R	Y OF ANTENNA OFFSET (KM)
46-65	D20.14	R	Z OF ANTENNA OFFSET (KM)

RECORD #
5

COLUMN	FORMAT	TYPE	DESCRIPTION
1-5	I5	I	ANTENNA ID = 2
6-25	D20.14	R	X OF ANTENNA OFFSET (KM)
26-45	D20.14	R	Y OF ANTENNA OFFSET (KM)
46-65	D20.14	R	Z OF ANTENNA OFFSET (KM)

RECORD #
6

COLUMN	FORMAT	TYPE	DESCRIPTION
1-5	I5	I	ANTENNA ID = 3
6-25	D20.14	R	X OF ANTENNA OFFSET (KM)
26-45	D20.14	R	Y OF ANTENNA OFFSET (KM)
46-65	D20.14	R	Z OF ANTENNA OFFSET (KM)

RECORD #
7

COLUMN	FORMAT	TYPE	DESCRIPTION
1-5	I5	I	ANTENNA ID = 4
6-25	D20.14	R	X OF ANTENNA OFFSET (KM)
26-45	D20.14	R	Y OF ANTENNA OFFSET (KM)
46-65	D20.14	R	Z OF ANTENNA OFFSET (KM)

14.13 Block II R Body-fixed Forces (unit/IUNIT/20)

Block II R Body-fixed Forces

Azimuth(degrees) D15.6	Elevation(degrees) D15.6	stfx(newtons) D15.6	stfy(newtons) D15.6	stfz(newtons) D15.6
0.000000D+00	-4.000000D+00	1.943110D-08	6.049400D-06	-1.096780D-04
0.000000D+00	-3.000000D+00	5.340160D-09	4.468250D-06	-1.090890D-04
0.000000D+00	-2.000000D+00	7.849870D-09	2.859420D-06	-1.087050D-04
0.000000D+00	-1.000000D+00	2.988930D-09	1.254930D-06	-1.080920D-04
0.000000D+00	0.000000D+00	9.633840D-10	-3.326060D-07	-1.073250D-04
0.000000D+00	1.000000D+00	-5.489280D-09	-1.923410D-06	-1.080080D-04
0.000000D+00	2.000000D+00	-1.428250D-08	-3.531440D-06	-1.086900D-04
0.000000D+00	3.000000D+00	-1.927720D-08	-5.155800D-06	-1.091130D-04
0.000000D+00	4.000000D+00	-3.040020D-08	-6.732390D-06	-1.097770D-04
1.000000D+00	-4.000000D+00	-1.856160D-06	6.050110D-06	-1.096670D-04
1.000000D+00	-3.000000D+00	-1.856820D-06	4.468450D-06	-1.091290D-04
1.000000D+00	-2.000000D+00	-1.855510D-06	2.858220D-06	-1.087060D-04
1.000000D+00	-1.000000D+00	-1.852190D-06	1.253690D-06	-1.080940D-04
1.000000D+00	0.000000D+00	-1.850250D-06	-3.333940D-07	-1.081040D-04
1.000000D+00	1.000000D+00	-1.851560D-06	-1.930790D-06	-1.080470D-04
1.000000D+00	2.000000D+00	-1.868160D-06	-3.545390D-06	-1.087110D-04
1.000000D+00	3.000000D+00	-1.890460D-06	-5.181540D-06	-1.097990D-04
1.000000D+00	4.000000D+00	-1.890870D-06	-6.764330D-06	-1.103880D-04
3.640000D+02	-4.000000D+00	-7.540850D-06	6.074900D-06	-1.095450D-04
3.640000D+02	-3.000000D+00	-7.542070D-06	4.511270D-06	-1.098630D-04
3.640000D+02	-2.000000D+00	-7.535390D-06	2.889320D-06	-1.102210D-04
3.640000D+02	-1.000000D+00	-7.535610D-06	1.268450D-06	-1.104210D-04
3.640000D+02	0.000000D+00	-7.530330D-06	-3.341230D-07	-1.105510D-04
3.640000D+02	1.000000D+00	-7.537180D-06	-1.946330D-06	-1.105320D-04
3.640000D+02	2.000000D+00	-7.540450D-06	-3.582590D-06	-1.102960D-04
3.640000D+02	3.000000D+00	-7.542020D-06	-5.225100D-06	-1.099570D-04
3.640000D+02	4.000000D+00	-7.538960D-06	-6.794060D-06	-1.096610D-04
3.650000D+02	-4.000000D+00	-9.456610D-06	6.094770D-06	-1.099880D-04
3.650000D+02	-3.000000D+00	-9.457730D-06	4.514400D-06	-1.103140D-04
3.650000D+02	-2.000000D+00	-9.468970D-06	2.895040D-06	-1.107500D-04
3.650000D+02	-1.000000D+00	-9.470180D-06	1.277180D-06	-1.111000D-04
3.650000D+02	0.000000D+00	-9.476460D-06	-3.442300D-07	-1.113160D-04
3.650000D+02	1.000000D+00	-9.474040D-06	-1.957870D-06	-1.111290D-04
3.650000D+02	2.000000D+00	-9.482560D-06	-3.612660D-06	-1.108620D-04
3.650000D+02	3.000000D+00	-9.466900D-06	-5.243890D-06	-1.104610D-04
3.650000D+02	4.000000D+00	-9.464970D-06	-6.826880D-06	-1.101530D-04
-1.772000D-12	-2.970000D-07	-1.259000D-07		

14.14 IERS Solid-Earth Tabular Data (unit/ISETD/46)

2 0 0.30190 0.29525 (nominal; elastic; anelastic (real, imag) k20

dood#	deg/hr	τ	s	h	p	N'	ps	l	l'	F	D	Ω	δk_f^R	Aip	δk_f^I	Aop
55.565	0.00221	0	0	0	0	1	0	0	0	0	0	1	0.01347	16.6	-0.00541	-6.7
55.575	0.00441	0	0	0	0	2	0	0	0	0	0	2	0.01124	-0.1	-0.00488	0.1
56.554	0.04107	0	0	1	0	0	-1	0	-1	0	0	0	0.00547	-1.2	-0.00349	0.8
57.555	0.08214	0	0	2	0	0	0	0	0	-2	2	-2	0.00403	-5.5	-0.00315	4.3
57.565	0.08434	0	0	2	0	1	0	0	0	-2	2	-1	0.00398	0.1	-0.00313	-0.1
58.554	0.12320	0	0	3	0	0	-1	0	-1	-2	2	-2	0.00326	-0.3	-0.00296	0.2
63.655	0.47152	0	1	-2	1	0	0	1	0	0	-2	0	0.00101	-0.3	-0.00242	0.7
65.445	0.54217	0	1	0	-1	-1	0	-1	0	0	0	-1	0.00080	0.1	-0.00237	-0.2
65.455	0.54438	0	1	0	-1	0	0	-1	0	0	0	0	0.00080	-1.2	-0.00237	3.7
65.465	0.54658	0	1	0	-1	1	0	-1	0	0	0	1	0.00079	0.1	-0.00237	-0.2
65.655	0.55366	0	1	0	1	0	0	1	0	-2	0	-2	0.00077	0.1	-0.00236	-0.2
73.555	1.01590	0	2	-2	0	0	0	0	0	0	-2	0	-0.00009	0.0	-0.00216	0.6
75.355	1.08875	0	2	0	-2	0	0	-2	0	0	0	0	-0.00018	0.0	-0.00213	0.3
75.555	1.09804	0	2	0	0	0	0	0	0	-2	0	-2	-0.00019	0.6	-0.00213	6.3
75.565	1.10024	0	2	0	0	1	0	0	0	-2	0	-1	-0.00019	0.2	-0.00213	2.6
75.575	1.10245	0	2	0	0	2	0	0	0	-2	0	0	-0.00019	0.0	-0.00213	0.2
83.655	1.56956	0	3	-2	1	0	0	1	0	-2	-2	-2	-0.00065	0.1	-0.00202	0.2
85.455	1.64241	0	3	0	-1	0	0	-1	0	-2	0	-2	-0.00071	0.4	-0.00201	1.1
85.465	1.64462	0	3	0	-1	1	0	-1	0	-2	0	-1	-0.00071	0.2	-0.00201	0.5
93.555	2.11394	0	4	-2	0	0	0	0	0	-2	-2	-2	-0.00102	0.1	-0.00193	0.2
95.355	2.18679	0	4	0	-2	0	0	-2	0	-2	0	-2	-0.00106	0.1	-0.00192	0.1
		-1														

2 1 0.29470 0.29830 0.00144 (nominal; elastic; anelastic (real, imag) k21

dood#	deg/hr	τ	s	h	p	N'	ps	l	l'	F	D	Ω	δk_f^{el}	Ael	δk_f^{anel}	Aanel
135.645	13.39645	1	-2	0	1	-1	0	1	0	2	0	1	-0.00044	-0.1	-0.00045	-0.1
135.655	13.39866	1	-2	0	1	0	0	1	0	2	0	2	-0.00044	-0.7	-0.00046	-0.7
137.455	13.47151	1	-2	2	-1	0	0	-1	0	2	2	2	-0.00047	-0.1	-0.00049	-0.1
145.545	13.94083	1	-1	0	0	-1	0	0	0	2	0	1	-0.00081	-1.2	-0.00082	-1.3
145.555	13.94303	1	-1	0	0	0	0	0	0	2	0	2	-0.00081	-6.6	-0.00082	-6.7
153.655	14.41456	1	0	-2	1	0	0	1	0	2	-2	2	-0.00167	0.1	-0.00168	0.1
155.455	14.48741	1	0	0	-1	0	0	-1	0	2	0	2	-0.00193	0.4	-0.00193	0.4
155.655	14.49669	1	0	0	1	0	0	1	0	0	0	0	-0.00196	1.3	-0.00197	1.3
155.665	14.49890	1	0	0	1	1	0	1	0	0	0	1	-0.00197	0.2	-0.00198	0.3
157.455	14.56955	1	0	2	-1	0	0	-1	0	0	2	0	-0.00231	0.3	-0.00231	0.3
162.556	14.91787	1	1	-3	0	0	1	0	1	2	-2	2	-0.00834	-1.9	-0.00832	-1.9
163.545	14.95673	1	1	-2	0	-1	0	0	0	2	-2	1	-0.01114	0.5	-0.01111	0.5
163.555	14.95893	1	1	-2	0	0	0	0	0	2	-2	2	-0.01135	-43.3	-0.01132	-43.2
164.556	15.00000	1	1	-1	0	0	1	0	1	0	0	0	-0.01650	2.0	-0.01642	2.0
165.545	15.03886	1	1	0	0	-1	0	0	0	0	0	-1	-0.03854	-8.8	-0.03846	-8.8
165.555	15.04107	1	1	0	0	0	0	0	0	0	0	0	-0.04093	472.0	-0.04085	471.0
165.565	15.04328	1	1	0	0	1	0	0	0	0	0	1	-0.04365	68.3	-0.04357	68.2
165.575	15.04548	1	1	0	0	2	0	0	0	0	0	2	-0.04678	-1.6	-0.04670	-1.6
166.554	15.08214	1	1	1	0	0	-1	0	-1	0	0	0	0.23083	-20.8	0.22609	-20.4
167.555	15.12321	1	1	2	0	0	0	0	0	-2	2	-2	0.03051	-5.0	0.03027	-5.0
173.655	15.51259	1	2	-2	1	0	0	1	0	0	-2	0	0.00374	-0.5	0.00371	-0.5
175.455	15.58545	1	2	0	-1	0	0	-1	0	0	0	0	0.00329	-2.1	0.00325	-2.1
175.465	15.58765	1	2	0	-1	1	0	-1	0	0	0	1	0.00327	-0.4	0.00324	-0.4
183.555	16.05697	1	3	-2	0	0	0	0	0	0	-2	0	0.00198	-0.2	0.00195	-0.2
185.555	16.13911	1	3	0	0	0	0	0	0	-2	0	-2	0.00187	-0.7	0.00184	-0.6
185.565	16.14131	1	3	0	0	1	0	0	0	-2	0	-1	0.00187	-0.4	0.00184	-0.4
		-1														

2 2 0.29801 0.30102 0.00130 (nominal; elastic; anelastic (real, imag) k22

dood#	deg/hr	τ	s	h	p	N'	ps	l	l'	F	D	Ω	δk_f^R	Amp
245.655	28.43973	2	-1	0	1	0	0	1	0	2	0	2	0.00006	-0.3
255.555	28.98410	2	0	0	0	0	0	0	0	2	0	2	0.00004	-1.2
		-1												

ftp://maia.usno.navy.mil/conventions/chapter6/isc6.tex

14.15 Predicted Mean Pole Values (unit/MEANPO/48)

```
 -.700   179.400     0.862     3.217  1950.000
  Xmo      Ymo        Xmr       Ymr    Yo-ref. year
Predicted Mean Pole Values- Format(5f10.3)
```

14.16 Ocean Tide Values (Partial Listing) (unit/IOCFIL/45)

```
**Line of text**
MULTI-SATELLITE OCEAN TIDE SELECTION : TOPEX 3.0 FOR DIUR/SEMI & SELF-CONSISTENT EQUILIBRIUM FOR LP + TEG-2B
**nx1 nx2 nxt mxt**
2231924  30  30
**unused line**
0
**First six values unused then nxt-1=29 deformation coefficients**
 0.6378145000000E+07  0.1025000000000E+04  0.5974230000000E+25  0.4344000000000E+03 -0.1230000000000E-02
 0.0000000000000E+00 -0.3075000000000E+00 -0.1950000000000E+00 -0.1320000000000E+00 -0.1032000000000E+00 -0.8916666666667E-01
-0.8171039264055E-01 -0.7550000000000E-01 -0.7168568341226E-01 -0.6820000000000E-01 -0.6598006934454E-01 -0.6381245564559E-01
-0.6173208554894E-01 -0.5975418812791E-01 -0.5788336881686E-01 -0.5611852021255E-01 -0.5445554491728E-01 -0.5288888888889E-01
-0.5152965718034E-01 -0.5023692383148E-01 -0.4900764374167E-01 -0.4783846508377E-01 -0.4672594242301E-01 -0.4566666666667E-01
-0.4465734216676E-01 -0.4369483010918E-01 -0.4277617040408E-01 -0.4189858994911E-01 -0.4105950237258E-01 -0.4025650258465E-01
**Start of the first large data block (only a few Doodson's numbers given in this partial listing**
 65.445       1     0.2310000000000E-02 0.3614455318269 2E-01 0.1800000000000E+03 0.2990000000000E+00
 65.455MM     8    -0.3518000000000E-01 0.3629164699348 9E-01 0.0000000000000E+00 0.2990000000000E+00
 65.465       1     0.2290000000000E-02 0.3643874080428 5E-01 0.1800000000000E+03 0.2990000000000E+00
 65.655       1     0.1880000000000E-02 0.3691055568011 1E-01 0.1800000000000E+03 0.2990000000000E+00
 65.665       1     0.7700000000000E-03 0.3705764949090 7E-01 0.1800000000000E+03 0.2990000000000E+00
 75.345       1     0.1500000000000E-03 0.7243620017618 1E-01 0.1800000000000E+03 0.2990000000000E+00
 75.355       4    -0.2880000000000E-02 0.7258329398697 8E-01 0.0000000000000E+00 0.2990000000000E+00
 75.365       1     0.1900000000000E-03 0.7273038779777 4E-01 0.1800000000000E+03 0.2990000000000E+00
 75.555MF    12    -0.6663000000000E-01 0.7320220267359 9E-01 0.0000000000000E+00 0.2990000000000E+00
 75.565       8    -0.2762000000000E-01 0.7334929648439 6E-01 0.0000000000000E+00 0.2990000000000E+00
 75.575       2    -0.2580000000000E-02 0.7349639029519 3E-01 0.0000000000000E+00 0.2990000000000E+00
**Start of the second large data block (only a few Doodson's numbers given in this partial listing**
          0 065.445      2 0    0.40005073429657E-01   0.0000000000000E+00   0.40005073429657E-01   0.0000000000000E+00 0
          0 065.455MM    2 0   -0.60925475465600E+00   0.0000000000000E+00  -0.60925475465600E+00   0.0000000000000E+00 0
          0 065.455MM    3 0   -0.10135626028736E-01   0.0000000000000E+00  -0.10135626028736E-01   0.0000000000000E+00 0
          0 065.455MM    4 0    0.10281879545548E+00   0.0000000000000E+00   0.10281879545548E+00   0.0000000000000E+00 0
          0 065.455MM    5 0   -0.20203268186095E+00   0.0000000000000E+00  -0.20203268186095E+00   0.0000000000000E+00 0
          0 065.455MM    6 0    0.37648113424210E-01   0.0000000000000E+00   0.37648113424210E-01   0.0000000000000E+00 0
          0 065.455MM    7 0   -0.22684288477352E+00   0.0000000000000E+00  -0.22684288477352E+00   0.0000000000000E+00 0
          0 065.455MM    8 0    0.28000639165680E-01   0.0000000000000E+00   0.28000639165680E-01   0.0000000000000E+00 0
          0 065.455MM    9 0   -0.12260953184455E+00   0.0000000000000E+00  -0.12260953184455E+00   0.0000000000000E+00 0
          0 065.465      2 0    0.39658709157539E-01   0.0000000000000E+00   0.39658709157539E-01   0.0000000000000E+00 0
          0 065.655      2 0    0.32558241579115E-01   0.0000000000000E+00   0.32558241579115E-01   0.0000000000000E+00 0
          0 065.665      2 0    0.13335024476553E-01   0.0000000000000E+00   0.13335024476553E-01   0.0000000000000E+00 0
          0 075.345      2 0    0.25977320408868E-02   0.0000000000000E+00   0.25977320408868E-02   0.0000000000000E+00 0
          0 075.355      2 0   -0.49876455185027E-01   0.0000000000000E+00  -0.49876455185027E-01   0.0000000000000E+00 0
          0 075.355      3 0   -0.82974994209100E-03   0.0000000000000E+00  -0.82974994209100E-03   0.0000000000000E+00 0
          0 075.355      4 0    0.84172294176172E-02   0.0000000000000E+00   0.84172294176172E-02   0.0000000000000E+00 0
          0 075.355      5 0   -0.16539344052290E-01   0.0000000000000E+00  -0.16539344052290E-01   0.0000000000000E+00 0
          0 075.365      2 0    0.32904605851233E-02   0.0000000000000E+00   0.32904605851233E-02   0.0000000000000E+00 0
          0 075.555MF    2 0   -0.11539125725619E+01   0.0000000000000E+00  -0.11539125725619E+01   0.0000000000000E+00 0
          0 075.555MF    3 0   -0.19196610639417E-01   0.0000000000000E+00  -0.19196610639417E-01   0.0000000000000E+00 0
          0 075.555MF    4 0    0.19473610975550E+00   0.0000000000000E+00   0.19473610975550E+00   0.0000000000000E+00 0
          0 075.555MF    5 0   -0.38264461604307E+00   0.0000000000000E+00  -0.38264461604307E+00   0.0000000000000E+00 0
          0 075.555MF    6 0    0.71304542281270E-01   0.0000000000000E+00   0.71304542281270E-01   0.0000000000000E+00 0
          0 075.555MF    7 0   -0.42963449154234E+00   0.0000000000000E+00  -0.42963449154234E+00   0.0000000000000E+00 0
          0 075.555MF    8 0    0.53032478328860E-01   0.0000000000000E+00   0.53032478328860E-01   0.0000000000000E+00 0
          0 075.555MF    9 0   -0.23221924692445E+00   0.0000000000000E+00  -0.23221924692445E+00   0.0000000000000E+00 0
          0 075.555MF   10 0   -0.19755387687623E-01   0.0000000000000E+00  -0.19755387687623E-01   0.0000000000000E+00 0
          0 075.555MF    2 1   -0.13289929338427E-01   0.13117891418188E-02 -0.13289929338427E-01  -0.13117891418188E-02 0
          0 075.555MF    4 1    0.37452380128346E-02   0.11472754419063E-01  0.37452380128346E-02  -0.11472754419063E-01 0
          0 075.555MF    6 1   -0.18125668412762E-01  -0.35582928228958E-01 -0.18125668412762E-01   0.35582928228958E-01 0
          0 075.565      2 0   -0.47832905979530E+00   0.0000000000000E+00  -0.47832905979530E+00   0.0000000000000E+00 0
          0 075.565      3 0   -0.79575324307475E-02   0.0000000000000E+00  -0.79575324307475E-02   0.0000000000000E+00 0
          0 075.565      4 0    0.80723568234232E-01   0.0000000000000E+00   0.80723568234232E-01   0.0000000000000E+00 0
          0 075.565      5 0   -0.15861690372369E+00   0.0000000000000E+00  -0.15861690372369E+00   0.0000000000000E+00 0
          0 075.565      6 0    0.29557728617871E-01   0.0000000000000E+00   0.29557728617871E-01   0.0000000000000E+00 0
          0 075.565      7 0   -0.17809552238330E+00   0.0000000000000E+00  -0.17809552238330E+00   0.0000000000000E+00 0
          0 075.565      8 0    0.21983446667313E-01   0.0000000000000E+00   0.21983446667313E-01   0.0000000000000E+00 0
          0 075.565      9 0   -0.96261377758570E-01   0.0000000000000E+00  -0.96261377758570E-01   0.0000000000000E+00 0
          0 075.575      2 0   -0.44680991103253E-01   0.0000000000000E+00  -0.44680991103253E-01   0.0000000000000E+00 0
          0 075.575      4 0    0.75404346866156E-02   0.0000000000000E+00   0.75404346866156E-02   0.0000000000000E+00 0
**Start of the  last data block (only a few Doodson's numbers given in this partial listing**
**These constituents are used to replace constituents in the second data block if Doodson's number and n,m match**
TEG-2B    0 065.455MM    2 0   -0.53502000000000E+00 -0.98155000000000E-01 -0.53502000000000E+00 -0.98155000000000E-01 0
          0 065.455MM    3 0    0.23002500000000E+00 -0.11704000000000E+00  0.23002500000000E+00 -0.11704000000000E+00 0
          0 075.555MF    2 0   -0.10575950000000E+01 -0.49348000000000E+00 -0.10575950000000E+01 -0.49348000000000E+00 0
          0 075.555MF    3 0    0.15090000000000E+00 -0.15250500000000E+00  0.15090000000000E+00 -0.15250500000000E+00 0
**Note for example that in 065.455MM   2 0 the Doodson's number is 65.455, and (degree,order)=(n,m)=(2,0).**
**Comments such as this one do not appear in the ocean tide file**
```

ftp://ftp.csr.utexas.edu/pub/grav/OTIDES.TOPEX_3.0

14.17 Yaw Bias (unit/IBIASF/44)

```
Description:
If the line begins with GPS, the number attached to GPS
is a PRN.  The second entree of such line is the number
of different bias values for this PRN. Each GPS line is
followed by lines containing the bias information for
the satellite (one line per each value of the bias).
The numbers contained in those lines provide the year and
day when the particular bias went into effect, followed
by the bias type.

The nominal yaw bias is -sign(beta angle)*0.5 degree.
BiasType  0  no yaw bias
BiasType  1  normal bias, i.e., -sign(beta angle)*0.5 degree.
BiasType -1  anti-normal bias
BiasType +-2 fixed bias of +-0.5 degree
BiasType of abs value > 2 means set bias to BiasType.

Comment lines allowed provided line doesn't start with GPS.

Example of the yaw bias file for a couple of satellites.

GPS01  5
1980.  43.     0.
1994.  157.    1.
1995.  85.     2.
1995.  86.     -2.
1995.  267.    2.
GPS02  5
1980.  43.     0.
1993.  1.      1.
1995.  185.    2.
1995.  186.    -2.
1995.  321.    2.
```

14.18 Yaw Rate (unit/IYAWRT/43)

```
This format of this file was determined by JPL and as
such is different from most OMNIS files. Lines not
beginning with 'GPS' are considered comments except when
following a 'GPS' line. If the line begins with GPS the
number attached to GPS is a PRN. The second entree of
such line is the number of different yaw rate values for
this PRN. The third entree is the satellite's moment of
inertia. Each GPS line is followed by lines containing
the yaw rate information for the satellite (one line per
each value of yaw rate). The numbers contained in those
lines provide the year and day when the particular yaw
rate went into effect, plus the value of the yaw rate itself.
This file is maintained by the Ratavg program using data
obtained from JPL.

GPS 1  4       723.4
1997.  259.00  .116
1998.  66.00   .121
1998.  247.00  .113
1999.  74.00   .117
GPS 2  2       661.6
1999.  11.00   .135
1999.  74.00   .135
etc.
```

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APPENDIX

a_{ij} coefficients for UTC steps before Jan. 1, 1972

		A.1 - UTC		
Date	MJD	ai1	ai2	ai3
01-01-58	6204	-0.1618000D-01	0.9745798D-03	-0.3991597D-05
01-16-58	6219	0.1758511D-01	0.9402134D-03	-0.2288330D-05
02-06-58	6240	0.5616044D-01	0.8366484D-03	-0.1373626D-05
02-20-58	6254	0.8754361D-01	0.8397126D-03	0.4555439D-06
04-10-58	6303	0.1496110D+00	0.9429822D-03	-0.2131837D-05
05-31-58	6354	0.1926477D+00	0.8642607D-03	0.2022977D-05
06-12-58	6366	0.2233144D+00	0.9408642D-03	-0.1432435D-05
07-03-58	6387	0.2626176D+00	0.8881868D-03	0.1373626D-05
07-17-58	6401	0.2952014D+00	0.9007882D-03	-0.1810101D-06
10-23-58	6499	0.4016215D+00	0.8420982D-03	-0.2598091D-06
11-27-58	6534	0.4508753D+00	0.7688908D-03	0.2232538D-05
12-25-58	6562	0.4940484D+00	0.8903961D-03	0.4597416D-06
01-29-59	6597	0.5458328D+00	0.9169140D-03	0.4557703D-06
02-26-59	6625	0.5918885D+00	0.9479880D-03	-0.2074632D-06
08-02-59	6782	0.7350500D+00	0.1110000D-02	-0.5000000D-04
08-06-59	6786	0.7587371D+00	0.8704235D-03	-0.2258611D-06
08-27-59	6807	0.7969748D+00	0.8567179D-03	0.4459769D-06
10-01-59	6842	0.8472612D+00	0.8507324D-03	0.1213704D-05
11-05-59	6877	0.8983352D+00	0.9424176D-03	-0.3296703D-05
11-19-59	6891	0.9309792D+00	0.8688708D-03	0.4694539D-06
12-17-59	6919	0.9762495D+00	0.6729432D-03	0.1359837D-04
01-14-60	6947	0.1005776D+01	0.1276327D-02	-0.2546813D-07
06-30-60	7084	0.1179992D+01	0.1276583D-02	-0.7058271D-07
09-07-60	7184	0.1306970D+01	0.1251737D-02	0.2006400D-06
01-01-61	7300	0.1459942D+01	0.1290565D-02	-0.1259551D-07
04-20-61	7409	0.1600435D+01	0.1288668D-02	-0.4508204D-07
08-01-61	7512	0.1682730D+01	0.1297609D-02	-0.1611287D-07
12-17-61	7650	0.1861600D+01	0.1300000D-02	-0.1262726D-15
01-01-62	7665	0.1881260D+01	0.1121344D-02	0.9459211D-08
06-02-62	7817	0.2051849D+01	0.1116318D-02	0.2776893D-07
09-12-62	7919	0.2165931D+01	0.1120481D-02	0.5828122D-09
01-05-63	8034	0.2294768D+01	0.1115182D-02	0.6620223D-07
04-13-63	8132	0.2404605D+01	0.1124198D-02	-0.4265743D-08
08-14-63	8255	0.2542758D+01	0.1118567D-02	0.1036236D-07
11-01-63	8334	0.2731246D+01	0.1111958D-02	0.1542096D-06
01-06-64	8400	0.2805812D+01	0.1298018D-02	-0.1044524D-07
04-01-64	8486	0.3017345D+01	0.1293842D-02	0.1384168D-08
07-07-64	8583	0.3142858D+01	0.1294845D-02	-0.4026524D-07

a_{ij} coefficients for UTC steps before Jan. 1, 1972

			A.1 - UTC	
Date	MJD	ai1	ai2	ai3
09-01-64	8639	0.3315311D+01	0.1287598D-02	0.3103647D-06
10-01-64	8669	0.3355178D+01	0.1295054D-02	-0.1298027D-08
01-01-65	8761	0.3574267D+01	0.1294804D-02	0.4588893D-08
03-01-65	8820	0.3750694D+01	0.1297441D-02	-0.1153898D-07
07-01-65	8942	0.4008834D+01	0.1296214D-02	-0.8656292D-08
09-01-65	9004	0.4189151D+01	0.1296106D-02	-0.1562328D-08
12-09-65	9103	0.4317425D+01	0.1298000D-02	-0.1337040D-16
01-02-66	9127	0.4348522D+01	0.2591633D-02	0.1658375D-08
06-14-66	9290	0.4770985D+01	0.2594067D-02	-0.1855570D-07
09-25-66	9393	0.5038040D+01	0.2590933D-02	0.1948657D-07
12-01-66	9460	0.5210963D+01	0.2592218D-02	-0.4296287D-08
04-23-67	9603	0.5581668D+01	0.2593321D-02	-0.8806318D-08
08-11-67	9713	0.5866922D+01	0.2585386D-02	0.3478871D-07
11-30-67	9824	0.6154340D+01	0.2590120D-02	0.3398956D-07
02-01-68	9887	0.6217630D+01	0.2592005D-02	-0.3637691D-10
06-01-68	10008	0.6531262D+01	0.2591999D-02	0.6570560D-11
12-26-68	10216	0.7070398D+01	0.2592008D-02	-0.6112177D-10
05-18-69	10359	0.7441054D+01	0.2591984D-02	0.1292546D-09
09-07-69	10471	0.7731358D+01	0.2592001D-02	-0.5477020D-11
04-14-70	10690	0.8299006D+01	0.2592000D-02	0.2950026D-11
08-17-70	10815	0.8623006D+01	0.2592000D-02	-0.6895982D-16
12-08-70	10928	0.8915902D+01	0.2592000D-02	-0.2521662D-11
04-17-71	11058	0.9252862D+01	0.2592000D-02	0.2260392D-16
08-27-71	11190	0.9595006D+01	0.2591993D-02	0.8745209D-10
11-26-71	11281	0.9830878D+01	0.2592018D-02	-0.5629834D-16
01-01-72	11317	0.1003438D+02	0.0	0.0
05-25-72	11462	0.1003438D+02	0.0	0.0
07-01-72	11499	0.1103438D+02	0.0	0.0
12-01-72	11652	0.1103438D+02	0.0	0.0
01-01-73	11683	0.1203438D+02	0.0	0.0
05-22-73	11824	0.1203438D+02	0.0	0.0
09-12-73	11937	0.1203438D+02	0.0	0.0
12-23-73	12039	0.1203438D+02	0.0	0.0
01-01-74	12048	0.1303438D+02	0.0	0.0
05-10-74	12177	0.1303438D+02	0.0	0.0
07-12-74	12240	0.1303438D+02	0.0	0.0
08-12-74	12271	0.1303438D+02	0.0	0.0
08-15-74	12274	0.1303438D+02	0.0	0.0
08-22-74	12281	0.1303438D+02	0.0	0.0
08-28-74	12287	0.1303438D+02	0.0	0.0

a_{ij} coefficients for UTC steps before Jan. 1, 1972

			A.1 - UTC	
Date	MJD	ai1	ai2	ai3
09-05-74	12295	0.1303438D+02	0.0	0.0
09-11-74	12301	0.1303438D+02	0.0	0.0
09-23-74	12313	0.1303438D+02	0.0	0.0
09-27-74	12317	0.1303438D+02	0.0	0.0
09-30-74	12320	0.1303438D+02	0.0	0.0
10-05-74	12325	0.1303438D+02	0.0	0.0
10-09-74	12329	0.1303438D+02	0.0	0.0
01-01-75	12413	0.1403438D+02	0.0	0.0
04-08-75	12510	0.1403438D+02	0.0	0.0
05-03-75	12535	0.1403438D+02	0.0	0.0
05-07-75	12539	0.1403438D+02	0.0	0.0
05-13-75	12545	0.1403438D+02	0.0	0.0
05-24-75	12556	0.1403438D+02	0.0	0.0
05-31-75	12563	0.1403438D+02	0.0	0.0
06-05-75	12568	0.1403438D+02	0.0	0.0
06-12-75	12575	0.1403438D+02	0.0	0.0
06-18-75	12581	0.1403438D+02	0.0	0.0
06-28-75	12591	0.1403438D+02	0.0	0.0
07-01-75	12594	0.1403438D+02	0.2000000D-07	0.0
07-03-75	12596	0.1403439D+02	0.2020000D-07	0.0

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